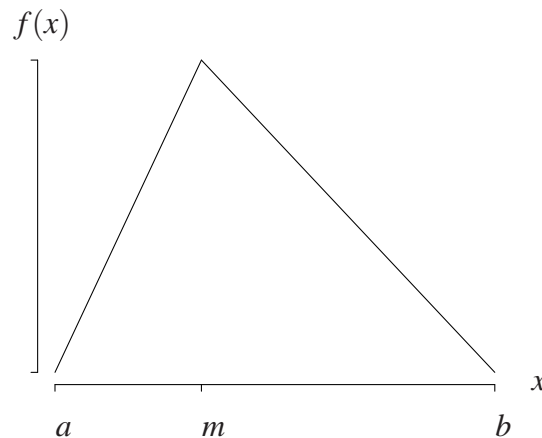


**Triangular distribution** (from <http://www.math.wm.edu/~leemis/chart/UDR/UDR.html>)

The shorthand  $X \sim \text{triangular}(a, m, b)$  is used to indicate that the random variable  $X$  has the triangular distribution with parameters  $a$ ,  $m$  and  $b$ . A triangular random variable  $X$  has probability density function

$$f(x) = \begin{cases} \frac{2(x-a)}{(b-a)(m-a)} & a < x < m \\ \frac{2(b-x)}{(b-a)(b-m)} & m \leq x < b. \end{cases}$$

The triangular distribution can be used as an approximate model when there are no data values. An expert familiar with the population specifies a minimum value  $a$ , a most likely value  $m$ , and a maximum value  $b$ . The probability density function is illustrated below.



The cumulative distribution function on the support of  $X$  is

$$F(x) = P(X \leq x) = \begin{cases} \frac{(x-a)^2}{(b-a)(m-a)} & a < x < m \\ 1 - \frac{(b-x)^2}{(b-a)(b-m)} & m \leq x < b. \end{cases}$$

The survivor function on the support of  $X$  is

$$S(x) = P(X \geq x) = \begin{cases} 1 - \frac{(x-a)^2}{(b-a)(m-a)} & a < x < m \\ \frac{(b-x)^2}{(b-a)(b-m)} & m \leq x < b. \end{cases}$$

The hazard function on the support of  $X$  is

$$h(x) = \frac{f(x)}{S(x)} = \begin{cases} \frac{2(a-x)}{ab-mb+ma+x^2-2ax} & a < x < m \\ \frac{2}{b-x} & m \leq x < b. \end{cases}$$

The moment generating function of  $X$  is

$$M(t) = E[e^{tX}] = -\frac{2(-me^{ta} + be^{ta} + be^{tm} + ae^{tm} - ae^{tb} + me^{tb})}{(a-m)(a-b)(m-b)t^2} \quad t > 0.$$

The characteristic function of  $X$  is

$$\phi(t) = E[e^{itX}] = \frac{2(-me^{ita} + be^{ita} + be^{itm} + ae^{itm} - ae^{itb} + me^{itb})}{(a-m)(a-b)(m-b)t^2} \quad t > 0.$$

The population mean, variance, skewness, and kurtosis of  $X$  are

$$E[X] = \frac{a+m+b}{3} \quad V[X] = \frac{a^2+m^2+b^2-ab-am-mb}{18}$$

$$E\left[\left(\frac{X-\mu}{\sigma}\right)^3\right] = \frac{\sqrt{2}(a+b-2m)(2a-b-m)(a-2b+m)}{5(a^2+b^2+m^2-ab-am-bm)^{3/2}} \quad E\left[\left(\frac{X-\mu}{\sigma}\right)^4\right] = \frac{12}{5}.$$

**APPL verification:** The APPL statements

```
X := TriangularRV(a, m, b);
CDF(X);
SF(X);
HF(X);
Mean(X);
Variance(X);
Skewness(X);
Kurtosis(X);
MGF(X);
```

verify the cumulative distribution function, survivor function, hazard function, population mean, variance, skewness, kurtosis, and moment generating function.