

Theorem The standard Cauchy distribution is a special case of the Student's t distribution when $n = 1$.

Proof The Student's t distribution has probability density function

$$f(x) = \frac{\Gamma\left(\frac{n+1}{2}\right)}{\sqrt{n\pi} \Gamma\left(\frac{n}{2}\right) \left(\frac{x^2}{n} + 1\right)^{(n+1)/2}} \quad -\infty < x < \infty.$$

When $n = 1$, this becomes

$$f(x) = \frac{\Gamma(1)}{\Gamma\left(\frac{1}{2}\right) \sqrt{\pi} (x^2 + 1)} \quad -\infty < x < \infty,$$

where $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$ and $\Gamma(1) = 1$. This reduces to

$$f(x) = \frac{1}{\pi (1 + x^2)} \quad -\infty < x < \infty,$$

which is the probability density function of the standard Cauchy distribution.

APPL verification: The APPL statements

`TRV(1);`

`StandardCauchyRV();`

yield the identical probability density functions

$$f(x) = \frac{1}{(1 + x^2) \pi} \quad -\infty < x < \infty.$$