

**Theorem** Random variates from the TSP distribution can be generated in closed-form by inversion.

**Proof** The  $TSP(a, b, m, n)$  distribution has cumulative distribution function

$$F(x) = \begin{cases} \frac{(x-a)^n(m-a)^{1-n}}{b-a} & a < x < m \\ -\frac{a+b(b-x)^n(b-m)^{-n}-b-m(b-x)^n(b-m)^{-n}}{b-a} & m \leq x < b. \end{cases}$$

Equating the cumulative distribution function to  $u$ , where  $0 < u < 1$  yields an inverse cumulative distribution function

$$F^{-1}(u) = \begin{cases} \left(\frac{u(b-a)}{(m-a)^{1-n}}\right)^{1/n} + a & 0 < u < \frac{m-a}{b-a} \\ -\left(\frac{(-u)(b-a)-a+b}{(b-m)^{-n+1}}\right)^{1/n} + b & \frac{m-a}{b-a} \leq u < 1. \end{cases}$$

So a closed-form variate generation algorithm using inversion for the  $TSP(a, b, m, n)$  distribution is

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generate  $U \sim U(0, 1)$ 
if ( $U < (m - a)/(b - a)$ ) then
     $X \leftarrow (u(b - a)/(m - a)^{1-n})^{1/n} + a$ 
else
     $X \leftarrow -((( -u)(b - a) - a + b)/(b - a)^{-n+1})^{1/n} + b$ 
endif
return( $X$ )

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