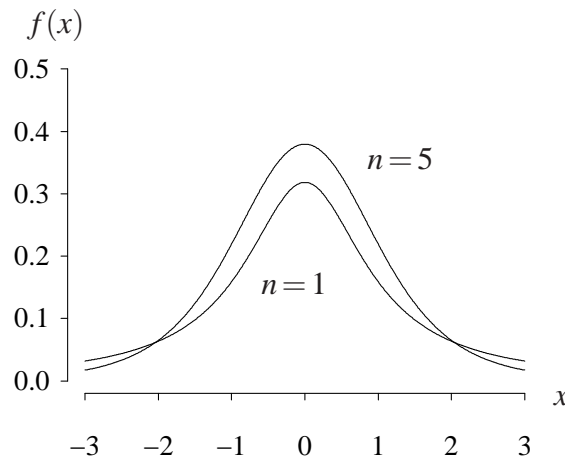


T distribution (from <http://www.math.wm.edu/~leemis/chart/UDR/UDR.html>)

The shorthand $X \sim t(n)$ is used to indicate that the random variable X has the t distribution with positive integer parameter n , known as the degrees of freedom. A t random variable X with n degrees of freedom has probability density function

$$f(x) = \frac{\Gamma((n+1)/2)(1+x^2/n)^{-(n+1)/2}}{\sqrt{n\pi}\Gamma(n/2)} \quad -\infty < x < \infty.$$

The t distribution arises in hypothesis tests concerning the comparison of (a) a sample mean to a standard, or (b) the difference between two means. The probability density function for $n = 1$ and $n = 5$ is illustrated below.



The cumulative distribution function on the support of X is

$$F(x) = P(X \leq x) = \int_{-\infty}^x \frac{\Gamma((n+1)/2)n^{n/2}(n+t^2)^{-(n+1)/2}}{\sqrt{\pi}\Gamma(n/2)} dt \quad -\infty < x < \infty.$$

The survivor function on the support of X is

$$S(x) = P(X \geq x) = \int_x^{\infty} \frac{\Gamma(\frac{1}{2} + \frac{n}{2})n^{n/2}(n+t^2)^{-(\frac{1}{2} + \frac{n}{2})}}{\sqrt{\pi}\Gamma(\frac{n}{2})} dt \quad -\infty < x < \infty.$$

The hazard function and cumulative hazard function on the support of X is mathematically intractable.

The inverse distribution function of X is mathematically intractable.

The median of X is 0 because $f(x)$ is an even function.

The moment generating function of X is mathematically intractable but can be determined in APPL

with statement at bottom of the page.

The population mean, variance, skewness, and kurtosis of X are

$$E[X] = 0 \quad \text{for } n > 1, \quad V[X] = \frac{n}{n-2} \quad \text{for } n > 2,$$
$$E \left[\left(\frac{X - \mu}{\sigma} \right)^3 \right] = 0 \quad \text{for } n > 3, \quad E \left[\left(\frac{X - \mu}{\sigma} \right)^4 \right] = \frac{3(n-2)}{n-4} \quad \text{for } n > 4.$$

APPL verification: The APPL statements

```
X := TRV(n);  
CDF(X);  
SF(X);  
Mean(X);  
Variance(X);  
Skewness(X);  
Kurtosis(X);  
MGF(X);
```

verify the cumulative distribution function, survivor function, hazard function, population mean, variance, skewness, kurtosis, and moment generating function.