Theorem [UNDER CONSTRUCTION: The scaling property seems to belong to the Inverse Gaussian rather than the Standard Wald] The standard Wald distribution has the scaling property. That is, if $X \sim$ standard Wald(λ) then $Y = kX$ also has the standard Wald distribution.

Proof [UNDER CONSTRUCTION: The scaling property seems to belong to the Inverse Gaussian rather than the Standard Wald] Let the random variable X have the standard Wald(λ) distribution with probability density function

$$f(x) = \sqrt{\frac{\lambda}{2\pi\, x^3}}\, e^{-\frac{\lambda(x-1)^2}{2x}} \qquad\qquad x > 0.$$

Let k be a positive, real constant. The transformation $Y = g(X) = kX$ is a 1–1 transformation from $\mathcal{X} = \{x \,|\, x > 0\}$ to $\mathcal{Y} = \{y \,|\, y > 0\}$ with inverse $X = g^{-1}(Y) = Y/k$ and Jacobian

$$\frac{dX}{dY} = \frac{1}{k}.$$

Therefore, by the transformation technique, the probability density function of Y is

$$\begin{aligned} f_Y(y) &= f_X(g^{-1}(y)) \left|\frac{dx}{dy}\right| \\ &= \sqrt{\frac{\lambda}{2\pi\,(y/k)^3}}\, e^{-\frac{\lambda(y/k-1)^2}{2(y/k)}} \left|\frac{1}{k}\right| \\ &= \sqrt{\frac{k\lambda}{2\pi\, y^3}}\, e^{-\frac{k\lambda(y-k)^2}{2k^2 y}} \qquad\qquad y > 0, \end{aligned}$$

which is the probability density function of a standard Wald($k\lambda$) random variable.

APPL failure: The APPL statements

```
assume(lambda > 0);
assume(k > 0);
X := [[x -> sqrt(lambda / (2 * Pi * x ^ 3)) * exp(-lambda * (x - 1) ^ 2 / (2 * x))],
      ["Continuous", "PDF"]];
g := [[x -> k * x], [0, infinity]];
Y := Transform(X, g);
```

do not yield the probability density function of a standard Wald random variable.