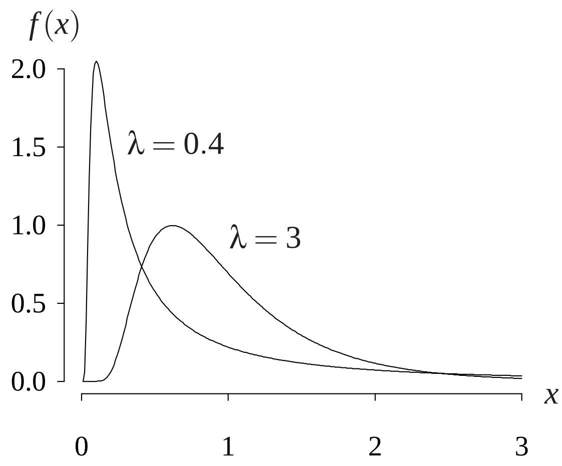
Standard Wald distribution (from `http://www.math.wm.edu/~leemis/chart/UDR/UDR.html`)
The shorthand $X \sim \text{standardWald}(\lambda)$ is used to indicate that the random variable X has the standard Wald distribution with parameter λ. A standard Wald random variable X with parameter λ has probability density function

$$f(x) = \sqrt{\frac{\lambda}{2\pi x^3}}\, e^{-\frac{\lambda(x-1)^2}{2x}} \qquad x > 0,$$

for $\lambda > 0$. The probability density function for two different values of λ is illustrated below.



The cumulative distribution function on the support of X is

$$F(x) = P(X \le x) = \int_0^x \sqrt{\frac{\lambda}{2\pi t^3}}\, e^{-\frac{\lambda(t-1)^2}{2t}}\, dt \qquad x > 0.$$

The survivor function, hazard function, inverse distribution, and cumulative hazard funtcions on the support of X are mathematically intractable. The moment generating function of X is

$$M(t) = E\left[e^{tX}\right] = e^{\lambda}\left(1 - \sqrt{1 - \frac{2t}{\lambda}}\right) \qquad t < \frac{\lambda}{2}.$$

The characteristic function of X is

$$\phi(t) = E\left[e^{itX}\right] = e^{\lambda}\left(1 - \sqrt{1 - \frac{2it}{\lambda}}\right) \qquad t < \frac{\lambda}{2}.$$

The population mean, variance, skewness, and kurtosis of X are

$$E[X] = 1 \qquad V[X] = \frac{1}{\lambda} \qquad E\left[\left(\frac{X-\mu}{\sigma}\right)^3\right] = \frac{3}{\sqrt{\lambda}} \qquad E\left[\left(\frac{X-\mu}{\sigma}\right)^4\right] = 3 + \frac{15}{\lambda}.$$

APPL verification: The APPL statements

```
X:=[[x -> sqrt(lambda / (Pi * 2 * x ^ 3)) * exp(-lambda(x - 1) ^ 2 / (2 * x))],
    [0,infinity], ["Continuous", "PDF"]];
CDF(X);
Mean(X);
Variance(X);
Skewness(X);
Kurtosis(X);
MGF(X);
```

verify the cumulative distribution function and moment generating function but fail to yield the expected values.