Theorem The standard uniform distribution has the variate generation property. That is, the inverse cumulative distribution function of a standard uniform random variable can be expressed in closed form.

Proof The standard uniform distribution has probability density function

$$f(x) = 1 \qquad\qquad 0 < x < 1,$$

and cumulative distribution function

$$F(x) = x \qquad\qquad 0 < x < 1.$$

Equating the cumulative distribution function to u, where $0 < u < 1$ yields an inverse cumulative distribution function

$$F^{-1}(u) = u \qquad\qquad 0 < u < 1.$$

A variate generation algorithm for the standard uniform distribution is

> generate $U \sim U(0, 1)$
> $X \leftarrow U$
> return(X)

APPL verification: The APPL statements

```
X := StandardUniformRV();
IDF(X);
```

produced the inverse cumulative distribution function

$$F^{-1}(u) = u \qquad\qquad 0 < u < 1.$$