Theorem The standard uniform distribution has the variate generation property. That is, the inverse cumulative distribution function of a standard uniform random variable can be expressed in closed form.

Proof The standard uniform distribution has probability density function

\[ f(x) = 1 \quad 0 < x < 1, \]

and cumulative distribution function

\[ F(x) = x \quad 0 < x < 1. \]

Equating the cumulative distribution function to \( u \), where \( 0 < u < 1 \) yields an inverse cumulative distribution function

\[ F^{-1}(u) = u \quad 0 < u < 1. \]

A variate generation algorithm for the standard uniform distribution is

\[
\begin{align*}
generate \ U \sim U(0,1) \\
X \leftarrow U \\
return(X)
\end{align*}
\]

APPL verification: The APPL statements

\[ X := \text{StandardUniformRV}(); \]
\[ \text{IDF}(X); \]

produced the inverse cumulative distribution function

\[ F^{-1}(u) = u \quad 0 < u < 1. \]