**Theorem** If $X \sim U(0, 1)$, then $Y = a + (b - a)X$ has the $U(a, b)$ distribution.

**Proof** Let the random variable $X \sim U(0, 1)$. The probability density function of $X$ is

$$f_X(x) = 1 \quad 0 < x < 1.$$  

The transformation $Y = g(X) = a + (b - a)X$ is a 1–1 transformation from $\mathcal{X} = \{x \mid 0 < x < 1\}$ to $\mathcal{Y} = \{y \mid a < y < b\}$ with inverse $X = g^{-1}(Y) = (Y - a)/(b - a)$ and Jacobian

$$\frac{dX}{dY} = \frac{1}{b - a}.$$  

Therefore, by the transformation technique, the probability density function of $Y$ is

$$f_Y(y) = f_X(g^{-1}(y)) \left| \frac{dx}{dy} \right| = \frac{1}{1/(b - a)} \cdot \frac{1}{b - a} = \frac{1}{b - a} \quad a < y < b,$$

which is the probability density function of a $U(a, b)$ random variable.