Theorem If $X \sim U(0,1)$, then $Y = a + (b-a)X$ has the $U(a,b)$ distribution.

Proof Let the random variable $X \sim U(0,1)$. The probability density function of X is

$$f_X(x) = 1 \qquad\qquad 0 < x < 1.$$

The transformation $Y = g(X) = a+(b-a)X$ is a 1–1 transformation from $\mathcal{X} = \{x \,|\, 0 < x < 1\}$ to $\mathcal{Y} = \{y \,|\, a < y < b\}$ with inverse $X = g^{-1}(Y) = (Y-a)/(b-a)$ and Jacobian

$$\frac{dX}{dY} = \frac{1}{b-a}.$$

Therefore, by the transformation technique, the probability density function of Y is

$$\begin{aligned} f_Y(y) &= f_X(g^{-1}(y)) \left|\frac{dx}{dy}\right| \\ &= 1(1/(b-a)) \\ &= \frac{1}{b-a} \qquad\qquad a < y < b, \end{aligned}$$

which is the probability density function of a $U(a,b)$ random variable.