Theorem The difference of two independent standard uniform random variables has the standard trianglular distribution.

Proof Let X_1 and X_2 be independent $U(0, 1)$ random variables. Let $Y = X_1 - X_2$. The joint probability density function of X_1 and X_2 is

$$f_{X_1,X_2}(x_1, x_2) = 1 \qquad 0 < x_1 < 1;\ 0 < x_2 < 1.$$

Using the cumulative distribution function technique, the cumulative distribution function of Y is

$$\begin{aligned}
F_Y(y) &= P(Y \le y) \\
&= P(X_1 - X_2 \le y) \\
&= \begin{cases} \int_0^{1+y} \int_{x_1-y}^{1} 1\, dx_2\, dx_1 & -1 < y < 0 \\ 1 - \int_y^1 \int_0^{x_1-y} 1\, dx_2\, dx_1 & 0 \le y < 1 \end{cases} \\
&= \begin{cases} y^2/2 + y + 1/2 & -1 < y < 0 \\ -y^2/2 + y + 1/2 & 0 \le y < 1. \end{cases}
\end{aligned}$$

Differentiating with respect to y yields the probability density function

$$f_Y(y) = \begin{cases} y + 1 & -1 < y < 0 \\ 1 - y & 0 \le y < 1 \end{cases}$$

which is the probability density function of the standard triangular distribution.

APPL verification: The APPL statements

```
X := UniformRV(0, 1);
Y := UniformRV(0, 1);
Difference(X, Y);
```

confirm the result.