

Theorem If X_1, X_2, \dots, X_n are mutually independent and identically distributed $U(0, 1)$ random variables, then $Y = \max\{X_1, X_2, \dots, X_n\}$ has the standard power distribution.

Proof Let the mutually independent random variables X_1, X_2, \dots, X_n each have the $U(0, 1)$ distribution. Let $Y = \max\{X_1, X_2, \dots, X_n\}$. Using an order statistic result, the probability density function of Y is

$$\begin{aligned} f_Y(y) &= \frac{n!}{(n-1)!(n-n)!} F(y)^{n-1} f(y) [1 - F(y)]^{n-n} \\ &= \frac{n!}{(n-1)!} y^{n-1} \\ &= ny^{n-1} \quad 0 < y < 1. \end{aligned}$$

which is the probability density function of the standard power distribution with $\beta = n$.

APPL verification: The APPL statements

```
X := StandardUniformRV();  
Y := OrderStat(X, n, n);
```

yield the probability density function of a standard power(n) random variable.