Theorem If $X \sim U(0,1)$, then $Y = X^{1/\beta}$ has the standard power(β) distribution, where $\beta > 1$.

Proof Let the random variable X have the standard uniform distribution with probability density function

$$f_X(x) = 1 \qquad\qquad 0 < x < 1.$$

The transformation $Y = g(X) = X^{1/\beta}$ is a 1–1 transformation from $\mathcal{X} = \{x \,|\, 0 < x < 1\}$ to $\mathcal{Y} = \{y \,|\, 0 < y < 1\}$ with inverse $X = g^{-1}(Y) = Y^{\beta}$ and Jacobian

$$\frac{dX}{dY} = \beta Y^{\beta-1}.$$

Therefore, by the transformation technique, the probability density function of Y is

$$\begin{aligned} f_Y(y) &= f_X(g^{-1}(y)) \left|\frac{dx}{dy}\right| \\ &= 1 \left|\beta y^{\beta-1}\right| \\ &= \beta y^{\beta-1} \qquad\qquad 0 < y < 1, \end{aligned}$$

which is the probability density function of a standard power(β) random variable.

APPL verification: The APPL statements

```
assume(beta > 1);
X := StandardUniformRV();
g := [[x -> x ^ (1 / beta)], [0, infinity]];
Y := Transform(X, g);
```

yield the probability density function of a standard power(β) random variable

$$f_Y(y) = \beta Y^{\beta-1} \qquad\qquad 0 < y < 1.$$