Theorem A standard uniform random variable $X$ can be transformed to a Pareto random variable $Y$ through the transformation

$$Y = \lambda X^{-1/\kappa},$$

where $\lambda$ and $\kappa$ are positive.

Proof Let the random variable $X$ have the standard uniform distribution with probability density function

$$f_X(x) = 1 \quad 0 < x < 1.$$

The transformation $Y = g(X) = \lambda X^{-1/\kappa}$ is a 1–1 transformation from $\mathcal{X} = \{x \mid 0 < x < 1\}$ to $\mathcal{Y} = \{y \mid y > 0\}$ with inverse $X = g^{-1}(Y) = (\lambda/y)^\kappa$ and Jacobian

$$\frac{dX}{dY} = \kappa \lambda^\kappa Y^{-\kappa - 1}.$$

Therefore, by the transformation technique, the probability density function of $Y$ is

$$f_Y(y) = f_X(g^{-1}(y)) \left| \frac{dx}{dy} \right| = (1) \left| \kappa \lambda^\kappa y^{-\kappa - 1} \right| = \frac{\kappa \lambda^\kappa}{y^{\kappa + 1}} \quad y > 0,$$

which is the probability density function of a Pareto random variable.