

Theorem A standard uniform random variable X can be transformed to a log logistic random variable Y through the transformation

$$Y = \frac{1}{\lambda} \left(\frac{1-X}{X} \right)^{1/\kappa},$$

where λ and κ are positive.

Proof Let the random variable X have the standard uniform distribution with probability density function

$$f_X(x) = 1 \quad 0 < x < 1.$$

The transformation $Y = g(X) = \frac{1}{\lambda} \left(\frac{1-X}{X} \right)^{1/\kappa}$ is a 1-1 transformation from $\mathcal{X} = \{x \mid 0 < x < 1\}$ to $\mathcal{Y} = \{y \mid y > 0\}$ with inverse $X = g^{-1}(Y) = \frac{1}{1+(\lambda Y)^\kappa}$ and Jacobian

$$\frac{dX}{dY} = \frac{-\kappa \lambda^\kappa Y^{\kappa-1}}{[1 + (\lambda Y)^\kappa]^2}.$$

Therefore by the transformation technique, the probability density function of Y is

$$\begin{aligned} f_Y(y) &= f_X(g^{-1}(y)) \left| \frac{dx}{dy} \right| \\ &= (1) \left| \frac{-\kappa \lambda^\kappa y^{\kappa-1}}{[1 + (\lambda y)^\kappa]^2} \right| \\ &= \frac{\lambda \kappa (\lambda \kappa)^{\kappa-1}}{[1 + (\lambda y)^\kappa]^2} \quad y > 0, \end{aligned}$$

which is the probability density function of the log logistic distribution.

APPL verification: The APPL statements

```
X := StandardUniformRV();
assume(lambda > 0);
assume(kappa > 0);
g := [[x -> 1 / lambda * ((1 - x) / x) ^ (1 / kappa)], [0, 1]];
Y := Transform(X, g);
simplify(Y[1][1](y));
Z := LogLogisticRV(lambda, kappa);
```

yield equivalent the functional forms

$$f_Y(y) = \frac{\lambda \kappa (\lambda \kappa)^{\kappa-1}}{[1 + (\lambda y)^\kappa]^2} \quad y > 0,$$

for the random variables Y and Z , which verifies that the standard uniform distribution can be transformed to the log logistic distribution.