**Theorem** A standard uniform random variable \( X \) can be transformed to a log logistic random variable \( Y \) through the transformation

\[
Y = \frac{1}{\lambda} \left( \frac{1 - X}{X} \right)^{1/\kappa},
\]

where \( \lambda \) and \( \kappa \) are positive.

**Proof** Let the random variable \( X \) have the standard uniform distribution with probability density function

\[
f_X(x) = 1, \quad 0 < x < 1.
\]

The transformation \( Y = g(X) = \frac{1}{X} \left( \frac{1-X}{X} \right)^{1/\kappa} \) is a 1–1 transformation from \( \mathcal{X} = \{ x \mid 0 < x < 1 \} \) to \( \mathcal{Y} = \{ y \mid y > 0 \} \) with inverse \( X = g^{-1}(Y) = \frac{1}{1 + (\lambda Y)^\kappa} \) and Jacobian

\[
\frac{dX}{dY} = \frac{-\kappa \lambda Y^{\kappa - 1}}{[1 + (\lambda Y)^\kappa]^2}.
\]

Therefore by the transformation technique, the probability density function of \( Y \) is

\[
f_Y(y) = f_X(g^{-1}(y)) \left| \frac{dx}{dy} \right| = \frac{\lambda \kappa (\lambda \kappa)^{\kappa - 1}}{[1 + (\lambda y)^\kappa]^2} \quad y > 0,
\]

which is the probability density function of the log logistic distribution.

**APPL verification:** The APPL statements

\[
X := \text{StandardUniformRV}();
\]
\[
\text{assume}(\lambda > 0);
\]
\[
\text{assume}(\kappa > 0);
\]
\[
g := \{ [x \rightarrow 1 / \lambda \cdot ((1-x) / x) ^ (1 / \kappa)] \}, [0, 1];
\]
\[
Y := \text{Transform}(X, g);
\]
\[
\text{simplify}(Y[1][1](y));
\]
\[
Z := \text{LogLogisticRV}(\lambda, \kappa);
\]

yield equivalent the functional forms

\[
f_Y(y) = \frac{\lambda \kappa (\lambda \kappa)^{\kappa - 1}}{[1 + (\lambda y)^\kappa]^2} \quad y > 0,
\]

for the random variables \( Y \) and \( Z \), which verifies that the standard uniform distribution can be transformed to the log logistic distribution.