**Theorem** A standard uniform random variable $X$ can be transformed to a log logistic random variable $Y$ through the transformation

$$Y = \frac{1}{\lambda} \left( \frac{1 - X}{X} \right)^{1/\kappa},$$

where $\lambda$ and $\kappa$ are positive.

**Proof** Let the random variable $X$ have the standard uniform distribution with probability density function

$$f_X(x) = 1, \quad 0 < x < 1.$$  

The transformation $Y = g(X) = \frac{1}{\lambda} \left( \frac{1 - X}{X} \right)^{1/\kappa}$ is a 1–1 transformation from $\mathcal{X} = \{x \mid 0 < x < 1\}$ to $\mathcal{Y} = \{y \mid y > 0\}$ with inverse $X = g^{-1}(Y) = \frac{1}{1 + (\lambda Y)^{\kappa}}$ and Jacobian

$$\frac{dX}{dY} = \frac{-\kappa \lambda^\kappa Y^{\kappa-1}}{(1 + (\lambda Y)^{\kappa})^2}.$$  

Therefore by the transformation technique, the probability density function of $Y$ is

$$f_Y(y) = f_X(g^{-1}(y)) \left| \frac{dx}{dy} \right|$$

$$= (1) \left| \frac{-\kappa \lambda^\kappa y^{\kappa-1}}{(1 + (\lambda y)^{\kappa})^2} \right|$$

$$= \frac{\lambda \kappa (\lambda y)^{\kappa-1} \lambda^\kappa Y^{\kappa-1}}{(1 + (\lambda y)^{\kappa})^2}$$  

$y > 0,$

which is the probability density function of the log logistic distribution.

**APPL verification:** The APPL statements

```appl
X := StandardUniformRV();
assume(lambda > 0);
assume(kappa > 0);
g := [[x -> 1 / lambda * ((1 - x) / x) ^ (1 / kappa)], [0, 1]];
Y := Transform(X, g);
simplify(Y[1][1](y));
Z := LogLogisticRV(lambda, kappa);
```

yield equivalent the functional forms

$$f_Y(y) = \frac{\lambda \kappa (\lambda y)^{\kappa-1} \lambda^\kappa Y^{\kappa-1}}{(1 + (\lambda y)^{\kappa})^2} \quad y > 0,$$

for the random variables $Y$ and $Z$, which verifies that the standard uniform distribution can be transformed to the log logistic distribution.