Theorem If $X \sim U(0,1)$, then $Y = \frac{1}{\lambda}\ln\left[1+\left(\frac{X}{1-X}\right)^{1/\kappa}\right]$ has the logistic-exponential(λ,κ) distribution, where λ and κ are positive parameters.

Proof Let the random variable X have the standard uniform distribution with probability density function

$$f_X(x) = 1 \qquad\qquad 0 < x < 1.$$

The transformation $Y = g(X) = \frac{1}{\lambda}\ln\left[1+\left(\frac{X}{1-X}\right)^{1/\kappa}\right]$ is a 1–1 transformation from $\mathcal{X} = \{x \,|\, 0 < x < 1\}$ to $\mathcal{Y} = \{y \,|\, y > 0\}$ with inverse $X = g^{-1}(Y) = \frac{(e^{\lambda Y}-1)^\kappa}{1+(e^{\lambda Y}-1)^\kappa}$ and Jacobian

$$\frac{dX}{dY} = \frac{\lambda\kappa(e^{\lambda Y}-1)^{\kappa-1}e^{\lambda Y}}{(1+(e^{\lambda Y}-1)^\kappa)^2}.$$

Therefore, by the transformation technique, the probability density function of Y is

$$\begin{aligned} f_Y(y) &= f_X(g^{-1}(y))\left|\frac{dx}{dy}\right| \\ &= 1\left|\frac{\lambda\kappa(e^{\lambda y}-1)^{\kappa-1}e^{\lambda y}}{(1+(e^{\lambda y}-1)^\kappa)^2}\right| \\ &= \frac{\lambda\kappa(e^{\lambda y}-1)^{\kappa-1}e^{\lambda y}}{(1+(e^{\lambda y}-1)^\kappa)^2} \qquad\qquad y > 0, \end{aligned}$$

which is the probability density function of a logistic-exponential(λ,κ) random variable.

APPL verification: After simplification, the APPL statements

```
assume(lambda > 0);
assume(kappa > 0);
X := StandardUniformRV();
g := [[x -> (1 / lambda) * log(1 + (x / (1 - x)) ^ (1 / kappa))], [0, 1]];
Y := Transform(X, g);
```

yield the probability density function of a logistic-exponential(λ,κ) random variable

$$f_Y(y) = \frac{\lambda\kappa(e^{\lambda Y}-1)^{\kappa-1}e^{\lambda Y}}{(1+(e^{\lambda Y}-1)^\kappa)^2} \qquad\qquad y > 0.$$