**Theorem** Let $X$ have the standard uniform distribution. Then $Y = \frac{\ln[1-(\ln X)(\ln \kappa)/\delta]}{\ln \kappa}$ has the Gompertz ($\kappa, \delta$) distribution, where $\kappa$ and $\delta$ are positive parameters with $\kappa > 1$.

**Proof** Let the random variable $X$ have the standard uniform distribution with probability density function

$$f_X(x) = 1 \quad 0 < x < 1.$$ 

The transformation $Y = g(X) = \frac{\ln[1-(\ln X)(\ln \kappa)/\delta]}{\ln \kappa}$ is a 1–1 transformation from $X = \{x \mid 0 < x < 1\}$ to $Y = \{y \mid y > 0\}$ with inverse $X = g^{-1}(Y) = e^{\delta(1-\kappa Y)/\ln \kappa}$ and Jacobian

$$\frac{dX}{dY} = -\delta \kappa^y e^{\delta(1-\kappa Y)/\ln \kappa}.$$ 

Therefore, by the transformation technique, the probability density function of $Y$ is

$$f_Y(y) = f_X(g^{-1}(y)) \left| \frac{dx}{dy} \right|$$

$$= 1 \left| -\delta \kappa^y e^{\delta(1-\kappa Y)/\ln \kappa} \right|$$

$$= \delta \kappa^y e^{\delta(1-\kappa Y)/\ln \kappa}$$

$$= \delta \kappa^y e^{-\delta(\kappa^y-1)/\ln \kappa} \quad y > 0,$$

which is the probability density function of the Gompertz($\kappa, \delta$) distribution.

**APPL verification:** The APPL statements

```appl
assume(delta > 0);
assume(kappa > 1);
X := StandardUniformRV();
g := [[x -> log(1 - (log(x) * log(kappa)) / delta)) / log(kappa)], [0, 1]]; Y := Transform(X, g);
```

yield the probability density function of a Gompertz($\kappa, \delta$) random variable

$$f_Y(y) = \delta \kappa^y e^{-\delta(\kappa^y-1)/\ln \kappa} \quad y > 0.$$