Theorem If $X \sim U(0,1)$, then $Y = \left[\frac{\ln(1-\ln(1-X))}{\lambda}\right]^{1/\kappa}$ has the exponential power(λ, κ) distribution, where λ and κ are positive parameters.

Proof Let the random variable X have the standard uniform distribution with probability density function

$$f_X(x) = 1 \qquad\qquad 0 < x < 1.$$

The transformation $Y = g(X) = \left[\frac{\ln(1-\ln(1-X))}{\lambda}\right]^{1/\kappa}$ is a 1–1 transformation from $\mathcal{X} = \{x \mid 0 < x < 1\}$ to $\mathcal{Y} = \{y \mid y > 0\}$ with inverse $X = g^{-1}(Y) = 1 - e^{1-e^{\lambda Y^\kappa}}$ and Jacobian

$$\frac{dX}{dY} = \left(e^{1-e^{\lambda Y^\kappa}}\right) e^{\lambda Y^\kappa} \lambda \kappa Y^{\kappa-1}.$$

Therefore, by the transformation technique, the probability density function of Y is

$$\begin{aligned} f_Y(y) &= f_X(g^{-1}(y)) \left|\frac{dx}{dy}\right| \\ &= 1 \left|(e^{1-e^{\lambda y^\kappa}}) e^{\lambda y^\kappa} \lambda \kappa y^{\kappa-1}\right| \\ &= (e^{1-e^{\lambda y^\kappa}}) e^{\lambda y^\kappa} \lambda \kappa y^{\kappa-1} \qquad\qquad y > 0, \end{aligned}$$

which is the probability density function of the exponential power(λ, κ) distribution.

APPL verification: The APPL statements

```
assume(lambda > 0);
assume(kappa > 0);
X := StandardUniformRV();
g := [[x -> (ln(1 - ln(1 - x)) / lambda) ^ (1 / kappa)], [0, infinity]];
Y := Transform(X, g);
```

yield the probability density function of a exponential power(λ, κ) random variable

$$f_Y(y) = (e^{1-e^{\lambda y^\kappa}}) e^{\lambda y^\kappa} \lambda \kappa y^{\kappa-1} \qquad\qquad y > 0.$$