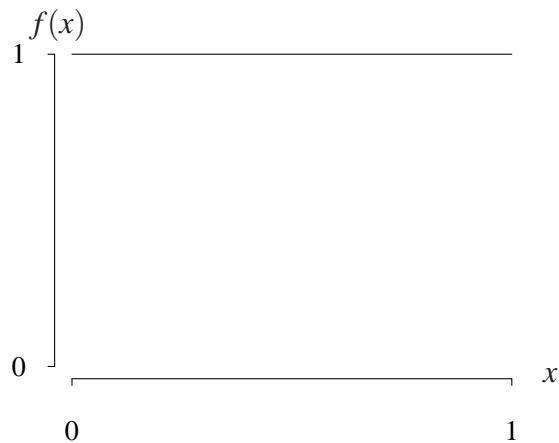


Standard uniform distribution (from <http://www.math.wm.edu/~leemis/chart/UDR/UDR.html>)

The shorthand $X \sim U(0, 1)$ is used to indicate that the random variable X has the standard uniform distribution with minimum 0 and maximum 1. A standard uniform random variable X has probability density function

$$f(x) = 1 \quad 0 < x < 1.$$

The standard uniform distribution is central to random variate generation. The probability density function is illustrated below.



The cumulative distribution function on the support of X is

$$F(x) = P(X \leq x) = x \quad 0 < x < 1.$$

The survivor function on the support of X is

$$S(x) = P(X \geq x) = 1 - x \quad 0 < x < 1.$$

The hazard function on the support of X is

$$h(x) = \frac{f(x)}{S(x)} = \frac{1}{1 - x} \quad 0 < x < 1.$$

The cumulative hazard function on the support of X is

$$H(x) = -\ln S(x) = -\ln(1 - x) \quad 0 < x < 1.$$

The inverse distribution function of X is

$$F^{-1}(u) = u \quad 0 < u < 1.$$

The median of X is $1/2$.

The moment generating function of X is

$$M(t) = \begin{cases} 1 & t = 0 \\ \frac{e^t - 1}{t} & t \neq 0 \end{cases}$$

The characteristic function of X is

$$\phi(t) = \begin{cases} 1 & t = 0 \\ \frac{e^{it} - 1}{it} & t \neq 0 \end{cases}$$

The population mean, variance, skewness and kurtosis of X are

$$E[X] = \frac{1}{2} \quad V[X] = \frac{1}{12} \quad E \left[\left(\frac{X - \mu}{\sigma} \right)^3 \right] = 0 \quad E \left[\left(\frac{X - \mu}{\sigma} \right)^4 \right] = \frac{9}{5}.$$

APPL verification: The APPL statements

```
X := UniformRV(0,1);  
Mean(X);  
Variance(X);  
Skewness(X);  
Kurtosis(X);  
MGF(X);
```

verify the population mean, variance, skewness, kurtosis, and moment generating function.