Theorem Random variates from the standard triangular distribution can be generated in closed-form by inversion.

Proof The standard triangular distribution has probability density function

$$f(x) = \begin{cases} x+1 & -1 < x < 0 \\ 1-x & 0 \le x < 1 \end{cases}$$

and cumulative distribution function

$$F(x) = \begin{cases} x^2/2 + x + 1/2 & -1 < x < 0 \\ -x^2/2 + x + 1/2 & 0 \le x < 1. \end{cases}$$

Equating the cumulative distribution function to u, where $0 < u < 1$ yields an inverse cumulative distribution function

$$F^{-1}(u) = \begin{cases} -1 + \sqrt{2u} & 0 < u < 1/2 \\ 1 - \sqrt{2 - 2u} & 1/2 \le u < 1. \end{cases}$$

So a closed-form variate generation algorithm using inversion for the standard triangular distribution is

generate $U \sim U(0, 1)$
if $(U < 1/2)$ then
    $X \leftarrow -1 + \sqrt{2U}$
else
    $X \leftarrow 1 - \sqrt{2 - 2U}$
endif
return(X)

APPL verification: The APPL statements

```
X := TriangularRV(-1, 0, 1);
CDF(X);
IDF(X);
```

produce the inverse distribution function of a standard triangular random variable.