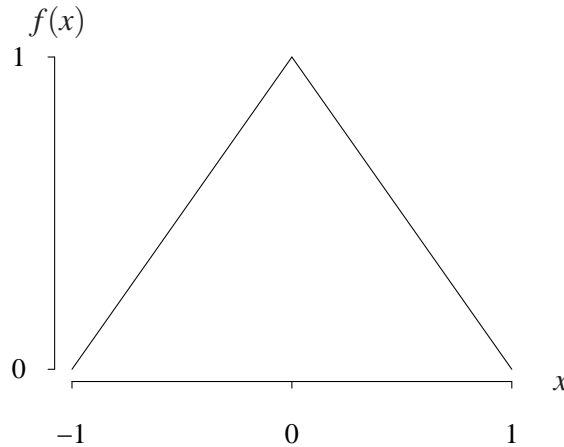


Standard triangular distribution (from <http://www.math.wm.edu/~leemis/chart/UDR/UDR.html>)

The shorthand $X \sim \text{triangular}(-1, 0, 1)$ is used to indicate that the random variable X has the standard triangular distribution. A standard triangular random variable X has the probability density function

$$f(x) = \begin{cases} x+1 & -1 < x < 0 \\ 1-x & 0 \leq x < 1. \end{cases}$$

The probability density function is illustrated below.



The cumulative distribution function on the support of X is

$$F(x) = \begin{cases} \frac{1}{2}x^2 + x + \frac{1}{2} & -1 < x < 0 \\ -\frac{1}{2}x^2 + x + \frac{1}{2} & 0 \leq x < 1. \end{cases}$$

The survivor function is

$$S(x) = \begin{cases} -\frac{1}{2}x^2 - x + \frac{1}{2} & -1 < x < 0 \\ \frac{1}{2}x^2 - x + \frac{1}{2} & 0 \leq x < 1. \end{cases}$$

The hazard function is

$$h(x) = \begin{cases} \frac{2(x+1)}{-x^2-2x+1} & -1 < x < 0 \\ \frac{2}{1-x} & 0 \leq x < 1. \end{cases}$$

The cumulative hazard function is

$$H(x) = \begin{cases} -\ln\left(-\frac{1}{2}x^2 - x + \frac{1}{2}\right) & -1 < x < 0 \\ -\ln\left(\frac{1}{2}x^2 - x + \frac{1}{2}\right) & 0 \leq x < 1. \end{cases}$$

The inverse distribution function of X is

$$F^{-1}(u) = \begin{cases} -1 + \sqrt{2u} & 0 < u < \frac{1}{2} \\ 1 - \sqrt{2-2u} & \frac{1}{2} \leq u < 1. \end{cases}$$

The median of X is 0. The mode of X is 0. The moment generating function of X is

$$M(t) = E[e^{tX}] = \begin{cases} 1 & t = 0 \\ \frac{e^{-t}-2+e^t}{t^2} & t \neq 0. \end{cases}$$

The population mean, variance, skewness, and kurtosis of X are

$$E[X] = 0 \quad V[X] = \frac{1}{6} \quad E\left[\left(\frac{X-\mu}{\sigma}\right)^3\right] = 0 \quad E\left[\left(\frac{X-\mu}{\sigma}\right)^4\right] = \frac{12}{5}.$$

An alternate definition of the standard triangular distribution exists. The alternate definition has a minimum of 0, a maximum of 1, and a parameter m , representing the mode.

APPL verification: The APPL statements

```
X := TriangularRV(-1,0,1);
CDF(X);
SF(X);
HF(X);
Mean(X);
Variance(X);
Skewness(X);
Kurtosis(X);
MGF(X);
```

verify the cumulative distribution function, survivor function, hazard function, population mean, variance, skewness, kurtosis, and moment generating function.