Theorem Random variates from the standard power(β) distribution can be generated in closed form by inversion.

Proof The standard power distribution has probability density function

$$f(x) = \beta x^{\beta-1} \qquad\qquad x \ge 0$$

and cumulative distribution function

$$F(x) = x^{\beta} \qquad\qquad x \ge 0.$$

Equating the cumulative distribution function to u, where $0 < u < 1$ yields an inverse distribution function

$$F^{-1}(u) = \sqrt[\beta]{u} \qquad\qquad 0 < u < 1.$$

So a closed-form variate generation algorithm using inversion for the standard power distribution is

generate $U \sim U(0,1)$
$X \leftarrow \sqrt[\beta]{u}$
return(X)

APPL verification: The APPL statements

```
assume(beta > 0);
X := [[x -> beta * x ^ (beta - 1)], [0, infinity], ["Continuous", "PDF"]];
CDF(X);
IDF(X);
```

yield identical forms of the cumulative distribution function and inverse distribution function as those given in the proof.