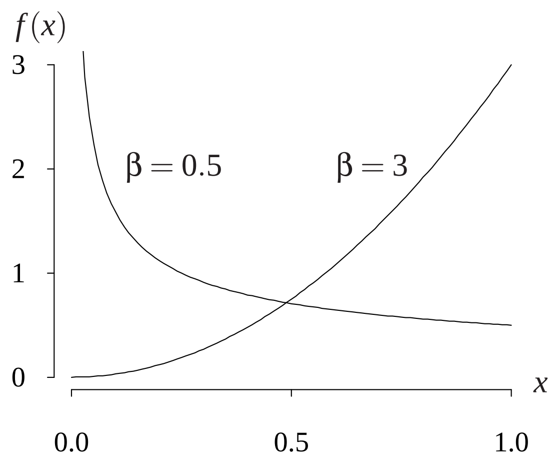
Standard power distribution (from http://www.math.wm.edu/~leemis/chart/UDR/UDR.html)
The shorthand $X \sim \text{power}(1, \beta)$ is used to indicate that the random variable X has the standard power distribution with shape parameter $\beta > 0$. A standard power random variable X with parameter β has probability density function

$$f(x) = \beta x^{\beta-1} \qquad 0 < x < 1.$$

The probability density function with two different values of β is illustrated below.



The cumulative distribution function on the support of X is

$$F(x) = P(X \le x) = x^{\beta} \qquad 0 < x < 1.$$

The survivor function on the support of X is

$$S(x) = P(X \ge x) = 1 - x^{\beta} \qquad 0 < x < 1.$$

The hazard function on the support of X is

$$h(x) = \frac{f(x)}{S(x)} = \frac{\beta x^{\beta-1}}{1 - x^{\beta}} \qquad 0 < x < 1.$$

The cumulative hazard function on the support of X is

$$H(x) = -\ln S(x) = -\ln(1 - x^{\beta}) \qquad 0 < x < 1.$$

The inverse distribution function of X is

$$F^{-1}(u) = u^{1/\beta} \qquad 0 < u < 1.$$

The median of X is

$$\left(\frac{1}{2}\right)^{1/\beta}.$$

The population mean, variance, skewness, and kurtosis of X are

$$E[X] = \frac{\beta}{\beta+1} \qquad V[X] = \frac{\beta}{(\beta+2)(\beta+1)^2}$$

$$E\left[\left(\frac{X-\mu}{\sigma}\right)^3\right] = \frac{2(1-\beta)\sqrt{\beta+2}}{(\beta+3)\sqrt{\beta}} \qquad E\left[\left(\frac{X-\mu}{\sigma}\right)^4\right] = \frac{3\left(3\beta^2-\beta+2\right)(\beta+2)}{\beta(\beta+3)(\beta+4)}.$$

APPL verification: The APPL statements

```
assume(beta > 0);
X := [[x -> beta * x ^ (beta - 1)], [0, 1], ["Continuous", "PDF"]];
Mean(X);
Variance(X);
Skewness(X);
Kurtosis(X);
MGF(X);
```

verify the population mean, variance, skewness, kurtosis, and moment generating function.