**Theorem** If \( X \sim N(0,1) \), then \( Y = \mu + \sigma X \) has the normal distribution with mean \( \mu \) and variance \( \sigma^2 \).

**Proof** Let the random variable \( X \sim N(0,1) \). The probability density function of \( X \) is

\[
f_X(x) = \frac{e^{-x^2/2}}{\sqrt{2\pi}} \quad -\infty < x < \infty.
\]

The transformation \( Y = \mu + \sigma X \) is a 1–1 transformation from \( \mathcal{X} = \{ x \mid -\infty < x < \infty \} \) to \( \mathcal{Y} = \{ y \mid -\infty < y < \infty \} \), with inverse \( X = (Y - \mu)/\sigma \) and Jacobian

\[
\frac{dX}{dY} = \frac{1}{\sigma}.
\]

Therefore, by the transformation technique, the probability density function of \( Y \) is

\[
f_Y(y) = f_X(g^{-1}(y)) \left| \frac{dx}{dy} \right| = \frac{e^{-(y-\mu)^2/2\sigma^2}}{\sqrt{2\pi}\sigma} \left| \frac{1}{\sigma} \right| = \frac{1}{\sqrt{2\pi}\sigma} e^{-(y-\mu)^2/2\sigma^2} \quad -\infty < x < \infty,
\]

which is the probability density function of a \( N(\mu, \sigma^2) \) random variable.

**APPL verification:** The APPL statements

\[
X := \text{StandardNormalRV}();
\]
\[
\text{assume}(\text{sigma} > 0);
\]
\[
g := [[x \rightarrow \mu + \sigma * x], [-\infty, \infty]];
\]
\[
Y := \text{Transform}(X, g);
\]

yield the probability density function of a \( N(\mu, \sigma^2) \) random variable.