Theorem If X is a standard normal random variable then $Y = |X|$ has the chi distribution with 1 degree of freedom.

Proof The cumulative distribution function of Y is

$$\begin{aligned} F_Y(y) &= P(Y \leq y) \\ &= P(|X| \leq y) \\ &= P(-y \leq X \leq y) \\ &= F_X(y) - F_X(-y) \\ &= 2F_X(y) \qquad\qquad y > 0 \end{aligned}$$

because of the symmetry of the standard normal distribution about 0. Differentiating with respect to y,

$$f_Y(y) = 2f_X(y) = \sqrt{\frac{2}{\pi}} e^{-y^2/2} \qquad\qquad y > 0,$$

which is the probability density function of a $\chi(1)$ random variable.

APPL Verification: The APPL statements

```
X := NormalRV(0, 1);
g := [[x -> -x, x -> x], [-infinity, 0, infinity]];
Z := Transform(X, g);
ChiRV(1);
```

confirm the result.