Standard normal distribution (from [http://www.math.wm.edu/~leemis/chart/UDR/UDR.html](http://www.math.wm.edu/~leemis/chart/UDR/UDR.html))

The shorthand $X \sim N(0, 1)$ is used to indicate that the random variable $X$ has the standard normal distribution. A standard normal random variable $X$ has probability density function

$$f(x) = \frac{e^{-x^2/2}}{\sqrt{2\pi}} \quad -\infty < x < \infty.$$ 

The standard normal random variable arises because a normal random variable with mean $\mu$ and variance $\sigma^2$ can be standardized by subtracting $\mu$, then dividing by $\sigma$. This means that only a single table is required for all calculations involving the normal distribution. The probability density function is illustrated below.

![Probability Density Function](chart.png)

The cumulative distribution function is

$$F(x) = \frac{1}{2} + \frac{1}{2} \text{erf} \left( \frac{x}{\sqrt{2}} \right) \quad -\infty < x < \infty$$

where

$$\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt \quad x > 0,$$

and $\text{erf}(-x) = -\text{erf}(x)$. The survivor function on the support of $X$ is

$$S(x) = \frac{1}{2} - \frac{1}{2} \text{erf} \left( \frac{x}{\sqrt{2}} \right) \quad -\infty < x < \infty.$$ 

The hazard function on the support of $X$ is

$$h(x) = -\frac{e^{-x^2/2}}{\sqrt{\pi} \left( -1 + \text{erf} \left( \frac{x}{\sqrt{2}} \right) \right)} \quad -\infty < x < \infty.$$
The cumulative hazard function on the support of $X$ is

$$H(x) = -\ln S(x) = \ln (2) - \ln \left(1 - \text{erf} \left(\frac{x}{\sqrt{2}}\right)\right) \quad -\infty < x < \infty.$$ 

The inverse distribution function of $X$ is

$$F^{-1}(u) = \sqrt{2}(\text{erf}^{-1}(2u - 1)) \quad 0 \leq u \leq 1.$$ 

The median and mode of $X$ are 0.

The moment generating function of $X$ is

$$M(t) = e^{t^2/2} \quad -\infty < t < \infty.$$ 

The characteristic function of $X$ is

$$\phi(t) = e^{-t^2/2} \quad -\infty < t < \infty.$$ 

The population mean, variance, skewness, and kurtosis of $X$ are

$$E[X] = 0 \quad V[X] = 1 \quad E \left[ \left(\frac{X - \mu}{\sigma}\right)^3 \right] = 0 \quad E \left[ \left(\frac{X - \mu}{\sigma}\right)^4 \right] = 3.$$ 

**APPL verification:** The APPL statements

```appl
X := StandardNormalRV();
CDF(X);
SF(X);
HF(X);
IDF(X);
Mean(X);
Variance(X);
Skewness(X);
Kurtosis(X);
MGF(X);
```

verify the cumulative distribution function, survivor function, hazard function, inverse distribution function, population mean, variance, skewness, kurtosis, and moment generating function.