**Theorem** The standard Cauchy distribution has the variate generation property.

**Proof** The standard Cauchy distribution has probability density function
\[ f(x) = \frac{1}{\pi (1 + x)^2} \quad -\infty < x < \infty, \]
so the cumulative distribution function of the standard Cauchy distribution is
\[ F(x) = \int_{-\infty}^{x} \frac{1}{\pi (1 + w)^2} dw = \frac{1}{2} + \frac{\arctan(x)}{\pi} \quad -\infty < x < \infty. \]
Equating the cumulative distribution function to \( u \), where \( 0 < u < 1 \) yields an inverse distribution function
\[ F^{-1}(u) = \tan \left( \pi \left( u - \frac{1}{2} \right) \right) \quad 0 < u < 1. \]
Simplifying using trigonometric identities,
\[ F^{-1}(u) = -\cot \left( \pi u \right) \quad 0 < u < 1. \]
So a random variate generation algorithm is
\[
\begin{align*}
generate & \quad U \sim U(0, 1) \\
X & \leftarrow -\cot(\pi U) \\
return & \quad (X)
\end{align*}
\]
**APPL verification:** The APPL statement
\[
\text{IDF(StandardCauchyRV());}
\]
returns the appropriate inverse distribution function.