**Theorem** The standard Cauchy distribution has the variate generation property.

**Proof** The standard Cauchy distribution has probability density function

$$f(x) = \frac{1}{\pi\left(1+x\right)^2} \qquad -\infty < x < \infty,$$

so the cumulative distribution function of the standard Cauchy distribution is

$$F(x) = \int_{-\infty}^{x} \frac{1}{\pi(1+w)^2} dw = \frac{1}{2} + \frac{\arctan(x)}{\pi} \qquad -\infty < x < \infty.$$

Equating the cumulative distribution function to $u$, where $0 < u < 1$ yields an inverse distribution function

$$F^{-1}(u) = \tan\left(\pi\left(u - \frac{1}{2}\right)\right) \qquad 0 < u < 1.$$

Simplifying using trigonometric identities,

$$F^{-1}(u) = -\cot\left(\pi u\right) \qquad 0 < u < 1.$$

So a random variate generation algorithm is

generate $U \sim U(0,1)$
$X \leftarrow -\cot(\pi U)$
return$(X)$

**APPL verification:** The APPL statement

```
IDF(StandardCauchyRV());
```

returns the appropriate inverse distribution function.