Theorem The standard Cauchy distribution has the scaling property. [UNDER CONSTRUCTION: It seems that the standard Cauchy distribution does NOT have the scaling property].

Proof [UNDER CONSTRUCTION: It seems that the standard Cauchy distribution does NOT have the scaling property]. Let the random variable X have the standard Cauchy distribution with probability density function

$$f_X(x) = \frac{1}{\pi(1+x^2)} \qquad -\infty < x < \infty.$$

Let $Y = g(X) = kX$, where k is a real number. The transformation $Y = g(X) = kX$ is a 1–1 transformation from $\mathcal{X} = \{x \mid -\infty < x < \infty\}$ to $\mathcal{Y} = \{y \mid -\infty < y < \infty\}$ with inverse $X = g^{-1}(Y) = \frac{Y}{k}$ and Jacobian

$$\frac{dX}{dY} = \frac{1}{k}.$$

Therefore by the transformation technique, the probability density function of Y is

$$\begin{aligned} f_Y(y) &= f_X(g^{-1}(y)) \left|\frac{dx}{dy}\right| \\ &= \frac{1}{\pi(1+(y/k)^2)} \left|\frac{1}{k}\right| \\ &= \frac{k^2}{\pi k(k^2+y^2)} \\ &= \frac{k}{\pi(k^2+y^2)} \qquad -\infty < y < \infty, \end{aligned}$$

which is the probability density function of the standard Cauchy distribution.

APPL verification: The APPL statements

```
X := StandardCauchyRV();
assume(k > 0);
g := [[x -> k * x], [-infinity, infinity]];
Y := Transform(X, g);
```

yield identical functional forms

$$f_Y(y) = \frac{k}{\pi\ (k^2+y^2)} \qquad -\infty < y < \infty$$

for the random variables X and Y, which verifies that the standard Cauchy distribution has the scaling property.