Theorem The inverse of a standard Cauchy random variable X is also standard Cauchy.

Proof Let the random variable X have the standard Cauchy distribution. The probability density function of X is

$$f_X(x) = \frac{1}{\pi\left(1+x^2\right)} \qquad\qquad -\infty < x < \infty.$$

Using the transformation technique, $Y = g(X) = 1/X$ is a 1–1 transformation from $\mathcal{X} = \{x \,|\, -\infty < x < \infty\}$ to $\mathcal{Y} = \{y \,|\, \infty < y < \infty\}$ with inverse $X = g^{-1}(y) = 1/Y$, and Jacobian $\frac{dX}{dY} = Y^{-2}$. Therefore, the probability density function of Y is

$$\begin{aligned} f_Y(y) &= f_X\left(g^{-1}(y)\right)\left|\frac{dx}{dy}\right| \\ &= \frac{1}{\pi\left[1+\left(\frac{1}{y^2}\right)\right]}\cdot\frac{1}{y^2} \\ &= \frac{1}{\pi\left(1+y^2\right)} \qquad\qquad -\infty < y < \infty, \end{aligned}$$

which is recognized as the standard Cauchy distribution probability density function.

APPL Verification: The APPL statements

```
X := StandardCauchyRV();
g := [[x -> 1 / x, x -> 1 / x], [-infinity, 0, infinity]];
Y := Transform(X, g);
```

verify the result.