Theorem If X is a standard Cauchy random variable then $Y = \ln|X|/\pi$ has the hyperbolic-secant distribution.

Proof Let X be a standard Cauchy random variable. The cumulative distribution function of X is

$$F_X(x) = \frac{\pi + 2\,\arctan(x)}{2\pi} \qquad -\infty < x < \infty.$$

The cumulative distribution function of $Y = \ln|X|/\pi$ is

$$\begin{aligned}
F_Y(y) &= P(Y \le y) \\
&= P(\ln|X|/\pi \le y) \\
&= P(|X| \le e^{\pi y}) \\
&= P(-e^{\pi y} \le X \le e^{\pi y}) \\
&= F_X(e^{\pi y}) - F_X(-e^{\pi y}) \\
&= \frac{\pi + 2\,\arctan(e^{\pi y})}{2\pi} - \frac{\pi + 2\,\arctan(-e^{\pi y})}{2\pi} \\
&= \frac{2}{\pi}\,\arctan(e^{\pi y}) \qquad -\infty < y < \infty,
\end{aligned}$$

which is the cumulative distribution function of a hyperbolic secant random variable.

APPL failure: The APPL statements

```
X := StandardCauchyRV();
g := [[x -> ln(-x) / Pi, x -> ln(x) / Pi], [-infinity, 0, infinity]];
Y := Transform(X, g);
CDF(Y);
```

yield the probability density function and cumulative distribution function of a hyperbolic secant random variable.