**Theorem** If $X$ has the standard Cauchy distribution, then $Y = a + \alpha X$ has the Cauchy distribution for $\alpha > 0$ and $-\infty < a < \infty$.

**Proof** Let the random variable $X$ have the standard Cauchy distribution. The probability density function of $X$ is

$$f_X(x) = \frac{1}{\pi [1 + x^2]} \quad -\infty < x < \infty.$$ 

Using the transformation technique, the transformation $Y = g(X) = a + \alpha X$ is a 1–1 transformation from $X = \{x \mid -\infty < x < \infty\}$ to $Y = \{y \mid \infty < y < \infty\}$ with inverse $X = g^{-1}(y) = \frac{y-a}{\alpha}$, and Jacobian $\frac{dX}{dY} = \frac{1}{\alpha}$. Therefore, the probability density function of $Y$ is

$$f_Y(y) = f_X\left(g^{-1}(y)\right) \left|\frac{dx}{dy}\right| = \frac{1}{\pi \left[1 + \left(\frac{y-a}{\alpha}\right)^2\right]} \cdot \frac{1}{\alpha} = \frac{1}{\alpha \pi \left(1 + \left((y-a)/\alpha\right)^2\right)} \quad -\infty < y < \infty,$$

which is recognized as the Cauchy distribution probability density function.

**APPL Verification:** The following APPL statements

```apl
assume(alpha > 0);
X := StandardCauchyRV();
g := [[x -> a + alpha * x], [-infinity, infinity]];
Transform(X, g);
```

yield the probability density function of the Cauchy distribution.