Standard Cauchy distribution (from [http://www.math.wm.edu/~leemis/chart/UDR/UDR.html](http://www.math.wm.edu/~leemis/chart/UDR/UDR.html))

The shorthand $X \sim \text{Cauchy}(1,0)$ is used to indicate that the random variable $X$ has the standard Cauchy distribution. A standard Cauchy random variable $X$ has probability density function

$$f(x) = \frac{1}{\pi (1+x^2)} \quad -\infty < x < \infty.$$ 

The probability density function is illustrated below.

![Standard Cauchy Distribution](image)

The cumulative distribution function on the support of $X$ is

$$F(x) = P(X \leq x) = \frac{\pi + 2 \arctan(x)}{2\pi} \quad -\infty < x < \infty.$$ 

The survivor function on the support of $X$ is

$$S(x) = P(X \geq x) = \frac{\pi - 2 \arctan(x)}{2\pi} \quad -\infty < x < \infty.$$ 

The hazard function on the support of $X$ is

$$h(x) = \frac{f(x)}{S(x)} = \frac{2}{(1+x^2)(\pi - 2 \arctan(x))} \quad -\infty < x < \infty.$$ 

The cumulative hazard function on the support of $X$ is

$$H(x) = -\ln S(x) = \ln(2) + \ln(\pi) - \ln(\pi - 2 \arctan(x)) \quad -\infty < x < \infty.$$ 

The inverse distribution function of $X$ is

$$F^{-1}(u) = -\cot(u \cdot \pi) \quad 0 < u < 1.$$ 

The median of $X$ is 0.
The moments of $X$ are undefined. It follows that the population mean, variance, skewness, and kurtosis of $X$ are also undefined.

**APPL verification:** The APPL statements

```apl
X := StandardCauchyRV( );
CDF(X);
SF(X);
HF(X);
CHF(X);
IDF(X);
```

verify the cumulative distribution function, survivor function, hazard function, and cumulative hazard function.