Theorem Random variates from the rectangular(n) distribution can be generated in closed-form by inversion.

Proof The rectangular(n) distribution has probability mass function

$$f(x) = \frac{1}{n+1} \qquad x = 0, 1, 2, \ldots, n$$

for some positive integer n. The cumulative distribution function is

$$F(x) = \frac{x+1}{n+1} \qquad x = 0, 1, 2, \ldots, n.$$

Equating the cumulative distribution function to u, where $0 < u < 1$, yields an inverse cumulative distribution function

$$F^{-1}(u) = \lfloor (n+1)u \rfloor \qquad 0 < u < 1.$$

So a closed-form variate generation algorithm using inversion for the rectangular(n) distribution is

generate $U \sim U(0, 1)$
$X \leftarrow \lfloor (n+1)u \rfloor$
return(X)

APPL verification: The APPL statements

```
X := [[x -> 1 / (n + 1)], [0 .. n], ["Discrete", "PDF"]];
CDF(X);
IDF(X);
```

produce the inverse distribution function of the rectangular random variable.