Theorem The Rayleigh distribution has the scaling property. That is, if $X \sim$ Rayleigh(α) then $kX \sim$ Rayleigh($k^2\alpha$) for a positive real constant k.

Proof Let X be a Rayleigh random variable with parameter α. Then, X has probability density function

$$f_X(x) = \frac{2x}{\alpha}\, e^{-x^2/\alpha} \qquad\qquad x > 0.$$

The transformation $Y = g(X) = kX$, for $k > 0$, is a 1–1 transformation from $\mathcal{X} = \{x | x > 0\}$ to $\mathcal{Y} = \{y | y > 0\}$ with inverse $X = g^{-1}(Y) = Y/k$. The Jacobian is $\frac{dX}{dY} = \frac{1}{k}$ Applying the transformation technique,

$$\begin{aligned} f_Y(y) &= f_X(g^{-1}(y)) \left|\frac{dX}{dY}\right| \\ &= f_X\left(\frac{y}{k}\right) \left|\frac{1}{k}\right| \\ &= \frac{2y}{k\alpha}\, e^{-y^2/(k^2\alpha)} \left|\frac{1}{k}\right| \\ &= \frac{2y}{k^2\alpha}\, e^{-y^2/(k^2\alpha)} \qquad\qquad y > 0, \end{aligned}$$

which is the probability density function of a Rayleigh($k^2\alpha$) random variable.

APPL verification: The APPL statements

```
assume(alpha > 0);
assume(k > 0);
X := [[x -> 2 * x / alpha * exp(-x ^ 2 / alpha)], [0, infinity],
       ["Continuous", "PDF"]];
g := [[x -> k * x], [0, infinity]];
Y := Transform(X, g);
```

verify the result.