Theorem Let $X_i \sim \text{Rayleigh}(\alpha_i)$ for $i = 1, 2, \ldots, n$ be mutually independent random variables. The minimum of $X_1, X_2, \ldots, X_n$ is also a Rayleigh random variable with parameter $1/\left(\sum_{i=1}^{n} 1/\alpha_i\right)$.

Proof The cumulative distribution function of the Rayleigh(α) random variable X is given by

$$\begin{aligned} F_X(x) &= \int_0^x \frac{2w}{\alpha} e^{-w^2/\alpha} dw \\ &= 1 - e^{-x^2/\alpha} \qquad\qquad x > 0. \end{aligned}$$

The cumulative distribution function of $Y = \min\{X_1, X_2, \ldots, X_n\}$.

$$\begin{aligned} F_Y(y) &= P(Y \le y) \\ &= 1 - P(Y \ge y) \\ &= 1 - P(\min\{X_1, X_2, \ldots, X_n\} \ge y) \\ &= 1 - P(X_1 \ge y, X_2 \ge y, \ldots, X_n \ge y) \\ &= 1 - [P(X_1 \ge y)\, P(X_2 \ge y) \ldots P(X_n \ge y)] \\ &= 1 - \left(e^{-y^2/\alpha_1}\right)\left(e^{-y^2/\alpha_2}\right) \cdots \left(e^{-y^2/\alpha_n}\right) \\ &= 1 - e^{\left(-y^2 \sum_{i=1}^{n} 1/\alpha_i\right)} \qquad\qquad y > 0. \end{aligned}$$

The probability density function is

$$f_Y(y) = 2y\left(\frac{1}{\alpha_1} + \frac{1}{\alpha_2} + \cdots + \frac{1}{\alpha_n}\right) e^{-y^2(1/\alpha_1 + 1/\alpha_2 + \cdots + 1/\alpha_n)} \qquad\qquad y > 0,$$

which is a Rayleigh$\left(1/\left(\sum_{i=1}^{n} 1/\alpha_i\right)\right)$ random variable.

APPL Verification: The APPL statements

```
assume(alpha1 > 0);
X1 := [[x -> 2 * x * exp(-x ^ 2 / alpha1) / alpha1], [0, infinity],
       ["Continuous", "PDF"]];
assume(alpha2 > 0);
X2 := [[x -> 2 * x * exp(-x ^ 2 / alpha2) / alpha2], [0, infinity],
       ["Continuous", "PDF"]];
Y  := Minimum(X1, X2);
```

yield the probability density function of the minimum of two independent Rayleigh random variables.