Theorem The square of a Rayleigh(α) random variable is an exponential(α) random variable.

Proof Let the random variable X have the Rayleigh distribution with probability density function

$$f_X(x) = \frac{2x}{\alpha}\, e^{-x^2/\alpha} \qquad\qquad x > 0,$$

for $\alpha > 0$. The transformation $Y = g(X) = X^2$ is a 1–1 transformation from $\mathcal{X} = \{x \,|\, x > 0\}$ to $\mathcal{Y} = \{y \,|\, y > 0\}$ with inverse $X = g^{-1}(Y) = \sqrt{y}$ and Jacobian

$$\frac{dX}{dY} = \frac{1}{2\sqrt{Y}}.$$

Therefore by the transformation technique, the probability density function of Y is

$$\begin{aligned} f_Y(y) &= f_X(g^{-1}(y)) \left|\frac{dx}{dy}\right| \\ &= \frac{2\sqrt{y}}{\alpha}\, e^{-\sqrt{y}^2/\alpha} \left|\frac{1}{2\sqrt{y}}\right| \\ &= \frac{1}{\alpha}\, e^{-y/\alpha} \qquad\qquad y > 0, \end{aligned}$$

which is the probability density function of the exponential distribution.

APPL verification: The APPL statements

```
assume(alpha > 0);
X := [[x -> 2 * x / alpha * exp(-x ^ 2 / alpha)], [0, infinity],
      ["Continuous", "PDF"]];
g := [[x -> x ^ 2], [0, infinity]];
Y := Transform(X, g);
```

yields the functional form

$$f_Y(y) = \frac{1}{\alpha} e^{-y/\alpha} \qquad\qquad y > 0$$

for the random variable Y.