Theorem The logarithm distribution is a special case of the power series$(c, A(c))$ distribution when $A(c) = -\ln(1-c)$.

Proof The power series$(c, A(c))$ distribution has probability mass function

$$f(x) = \frac{a_x c^x}{A(c)} \qquad x = 0, 1, 2, \ldots .$$

When $A(c) = -\ln(1-c)$,

$$f(x) = \frac{a_x c^x}{-\ln(1-c)} \qquad x = 0, 1, 2, \ldots .$$

Replacing c with $(1-c)$ and setting $a_x = 1/x$ we have

$$f(x) = \frac{-(1-c)^x}{x \ln c} \qquad x = 1, 2, 3, \ldots .$$

which is the probability mass function of the logarithm distribution.

APPL verification: The APPL statements

```
assume(c > 0);
assume(a[x] > 0);
X := [[x -> a[x] * c ^ x / A(c)], [0, infinity], ["Discrete", "PDF"]];
A := c -> -ln(1 - c);
c := 1 - c;
a[x] := 1 / x;
simplify(X[1][1](x));
```

yield the probability mass function of a logarithm(c) random variable.