Theorem The distribution of $\max\{X_1, X_2, \ldots, X_n\}$, where $X_i \sim \text{power}(\alpha, \beta_i)$, $i = 1, 2, \ldots, n$ are mutually independent random variables, has the power distribution.

Proof The power distribution has probability density function

$$f(x) = \frac{\beta x^{\beta-1}}{\alpha^\beta} \qquad\qquad 0 < x < \alpha$$

and cumulative distribution function

$$F(x) = \left(\frac{x}{\alpha}\right)^\beta \qquad\qquad 0 < x < \alpha$$

for $\alpha > 0$ and $\beta > 0$. Let $Y = \max\{X_1, X_2, \ldots, X_n\}$. The cumulative distribution function of Y is

$$\begin{aligned} F_Y(y) &= P(\max\{X_1, X_2, \ldots, X_n\} \le y) \\ &= P(X_1 \le y, X_2 \le y, \ldots, X_n \le y) \\ &= P(X_1 \le y)\, P(X_2 \le y) \ldots P(X_n \le y) \\ &= \left(\frac{y}{\alpha}\right)^{\beta_1} \left(\frac{y}{\alpha}\right)^{\beta_2} \cdots \left(\frac{y}{\alpha}\right)^{\beta_n} \\ &= \left(\frac{y}{\alpha}\right)^{\sum_{i=1}^n \beta_i} \qquad\qquad 0 < y < \alpha. \end{aligned}$$

This is recognized as the cumulative distribution function of a $\text{power}(\alpha, \sum_{i=1}^n \beta_i)$ random variable.