Theorem Random variates from the power(α, β) distribution can be generated in closed form by inversion.

Proof The power distribution has probability density function

$$f(x) = \frac{\beta x^{\beta-1}}{\alpha^{\beta}} \qquad x \geq 0$$

and cumulative distribution function

$$F(x) = \frac{x^{\beta}}{\alpha^{\beta}} \qquad x \geq 0.$$

Equating the cumulative distribution function to u, where $0 < u < 1$ yields an inverse distribution function

$$F^{-1}(u) = \alpha u^{1/\beta} \qquad 0 < u < 1.$$

So a closed-form variate generation algorithm using inversion for the power distribution is

generate $U \sim U(0, 1)$
$X \leftarrow \alpha u^{1/\beta}$
return(X)

APPL verification: The APPL statements

```
assume(alpha > 0);
assume(beta > 0);
X := [[x -> beta * x ^ (beta - 1) / alpha ^ beta], [0, infinity],
      ["Continuous", "PDF"]];
CDF(X);
IDF(X);
```

yield identical forms of the cumulative distribution function and inverse distribution function as those given in the proof.