**Theorem** The power distribution has the scaling property. That is, if $X \sim \text{power}(\alpha, \beta)$ then $Y = kX$ also has the power distribution.

**Proof** Let the random variable $X$ have the power$(\alpha, \beta)$ distribution with probability density function

$$f(x) = \frac{\beta x^{\beta-1}}{\alpha^\beta} \qquad 0 < x < \alpha.$$

Let $k$ be a positive, real constant. The transformation $Y = g(X) = kX$ is a 1–1 transformation from $\mathcal{X} = \{x \,|\, 0 < x < \alpha\}$ to $\mathcal{Y} = \{y \,|\, 0 < x < k\alpha\}$ with inverse $X = g^{-1}(Y) = Y/k$ and Jacobian

$$\frac{dX}{dY} = \frac{1}{k}.$$

Therefore, by the transformation technique, the probability density function of $Y$ is

$$\begin{aligned} f_Y(y) &= f_X(g^{-1}(y)) \left| \frac{dx}{dy} \right| \\ &= \frac{\beta (y/k)^{\beta-1}}{\alpha^\beta} \left| \frac{1}{k} \right| \\ &= \frac{\beta y^{\beta-1}}{(k\alpha)^\beta} \qquad 0 < y < k\alpha, \end{aligned}$$

which is the probability density function of a power$(k\alpha, \beta)$ random variable.

**APPL failure:** The APPL statements

```
assume(alpha > 0);
assume(beta > 0);
assume(k > 0);
X := [[beta * x ^ (beta - 1) / (alpha ^ beta)], [0, alpha],
      ["Continuous", "PDF"]];
g := [[x -> k * x], [0, alpha]];
Y := Transform(X, g);
```

do not yield the probability density function of a power$(k\alpha, \beta)$ random variable.