Theorem The binomial(n, p) distribution is a special case of the Polya(n, p, β) distribution in which $\beta = 0$.

Proof Let the random variable X have the Polya(n, p, β) distribution with probability mass function

$$f(x) = \frac{\binom{n}{x} \prod_{j=0}^{x-1} (p + j\beta) \prod_{k=0}^{n-x-1} (1 - p + k\beta)}{\prod_{i=0}^{n-1} (1 + i\beta)} \qquad x = 0, 1, 2, \ldots, n.$$

When $\beta = 0$,

$$\begin{aligned} f(x) &= \frac{\binom{n}{x} \prod_{j=0}^{x-1} p \prod_{k=0}^{n-x-1} (1 - p)}{\prod_{i=0}^{n-1} 1} \\ &= \binom{n}{x} p^x (1-p)^{n-x} \qquad x = 0, 1, 2, \ldots, n \end{aligned}$$

which is the probability mass function of the binomial(n, p) random variable.

APPL verification: The APPL statements

```
assume(n, posint);
assume(p > 0);
additionally(p < 1);
X := [[x -> ((n! / (x! * (n-x)!)) * product(p + j * beta, j = 0..x - 1)
     * product(1 - p + k * beta, k = 0 .. n - x - 1))
     / product(1 + i * beta, i = 0 .. n - 1)], [0, n], ["Discrete", "PDF"]];
beta := 0;
simplify(X[1][1](x));
```

yield the probability mass function of a binomial(n, p) random variable.