Theorem If $X_i \sim \text{Poisson}(\mu_i)$, for $i = 1, 2, \ldots, n$, and $X_1, X_2, \ldots, X_n$ are mutually independent random variables, then

$$\sum_{i=1}^{n} X_i \sim \text{Poisson}\left(\sum_{i=1}^{n} \mu_i\right).$$

Proof The moment generating function of X_i is

$$\begin{aligned}
M_{X_i}(t) &= E\left[e^{tX_i}\right] \\
&= \sum_{x=0}^{\infty} e^{tx}\,\frac{\mu_i^x e^{-\mu_i}}{x!} \\
&= e^{-\mu_i} \sum_{x=0}^{\infty} \frac{(\mu_i e^t)^x}{x!} \\
&= e^{-\mu_i} e^{\mu_i e^t} \\
&= e^{\mu_i (e^t - 1)}
\end{aligned}$$

for $-\infty < t < \infty$ and $i = 1, 2, \ldots, n$. Since the moment generating function of a sum of mutually independent random variables is the product of their moment generating functions,

$$\begin{aligned}
M_{X_1 + X_2 + \cdots + X_n}(t) &= \prod_{i=1}^{n} M_{X_i}(t) \\
&= \prod_{i=1}^{n} e^{\mu_i (e^t - 1)} \\
&= e^{\left(\sum_{i=1}^{n} \mu_i\right)(e^t - 1)}
\end{aligned}$$

for $-\infty < t < \infty$. This moment generating function is recognized as that of a Poisson random variable with mean $\sum_{i=1}^{n} \mu_i$.

APPL illustration: The APPL statements

```
X1 := PoissonRV(mu1);
X2 := PoissonRV(mu2);
simplify(MGF(X1) * MGF(X2));
```

yields the appropriate moment generating function

$$M_{X_1 + X_2}(t) = e^{(\mu_1 + \mu_2)(e^t - 1)} \qquad -\infty < t < \infty$$

for $n = 2$. The result holds for larger values of n by induction.