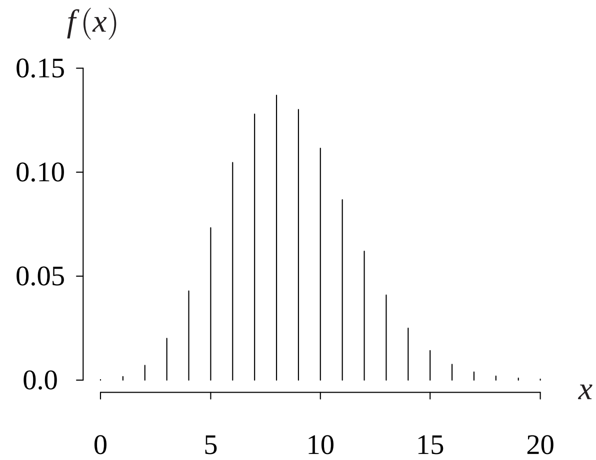
Poisson distribution (from `http://www.math.wm.edu/~leemis/chart/UDR/UDR.html`)
The shorthand $X \sim \text{Poisson}(\mu)$ is used to indicate that the random variable X has the Poisson distribution with positive parameter μ. A Poisson random variable X with scale parameter μ has probability mass function

$$f(x) = \frac{\mu^x e^{-\mu}}{x!} \qquad x = 0, 1, 2, \ldots.$$

The Poisson distribution can be used to model the number of events in an interval associated with a process that evolves randomly over space or time. Applications include the number of potholes over a stretch of highway, the number of typographical errors in a book, the number of customer arrivals in an hour, and the number of earthquakes in a decade. The Poisson distribution can also be used to approximate the binomial distribution when n is large and p is small. The probability mass function with $\mu = 8.56$ is illustrated below.



The cumulative distribution function on the support of X is

$$F(x) = P(X \le x) = \frac{\Gamma(x+1,\ \mu)}{\Gamma(x+1)} \qquad x = 0, 1, 2, \ldots.$$

The survivor function on the support of X is

$$S(x) = P(X \ge x) = \frac{\Gamma(x+1) - x\Gamma(x,\ \mu)}{x!} \qquad x = 0, 1, 2, \ldots.$$

The hazard function on the support of X is

$$h(x) = \frac{f(x)}{S(x)} = \frac{\mu^x e^{-\mu}}{\Gamma(x+1) - x\Gamma(x,\ \mu)} \qquad x = 0, 1, 2, \ldots.$$

The cumulative hazard function on the support of X is

$$H(x) = -\ln S(x) = -\ln\left[\frac{\Gamma(x+1) - x\Gamma(x,\ \mu)}{x!}\right] \qquad x = 0, 1, 2, \ldots.$$

The inverse distribution function of X is mathematically intractable but can run in APPL with statement at the bottom of the page.

The median, m, of X is approximately (see Wikipedia site)

$$m \approx \lfloor \mu + 1/3 - 0.02/\mu \rfloor .$$

The moment generating function of X is

$$M(t) = E\left[e^{tX}\right] = e^{\mu(e^t - 1)} \qquad -\infty < t < \infty$$

The characteristic function of X is

$$\phi(t) = E\left[e^{itX}\right] = e^{\mu(e^{it} - 1)} \qquad -\infty < t < \infty$$

The population mean, variance, skewness, and kurtosis of X are

$$E[X] = \mu \qquad V[X] = \mu \qquad E\left[\left(\frac{X-\mu}{\sigma}\right)^3\right] = \mu^{-1/2} \qquad E\left[\left(\frac{X-\mu}{\sigma}\right)^4\right] = 3 + \mu^{-1}.$$

APPL verification: The APPL statements

```
X := PoissonRV(mu);
CDF(X);
SF(X);
HF(X);
CHF(X);
IDF(X);
Mean(X);
Variance(X);
Skewness(X);
Kurtosis(X);
MGF(X);
```

verify the cumulative distribution function, survivor function, hazard function, cumulative hazard function, inverse distribution function, population mean, variance, skewness, kurtosis, and moment generating function.