Poisson distribution (from http://www.math.wm.edu/~leemis/chart/UDR/UDR.html)
The shorthand \( X \sim \text{Poisson}(\mu) \) is used to indicate that the random variable \( X \) has the Poisson distribution with positive parameter \( \mu \). A Poisson random variable \( X \) with scale parameter \( \mu \) has probability mass function

\[
f(x) = \frac{\mu^x e^{-\mu}}{x!}, \quad x = 0, 1, 2, \ldots.
\]

The Poisson distribution can be used to model the number of events in an interval associated with a process that evolves randomly over space or time. Applications include the number of potholes over a stretch of highway, the number of typographical errors in a book, the number of customer arrivals in an hour, and the number of earthquakes in a decade. The Poisson distribution can also be used to approximate the binomial distribution when \( n \) is large and \( p \) is small. The probability mass function with \( \mu = 8.56 \) is illustrated below.

The cumulative distribution function on the support of \( X \) is

\[
F(x) = P(X \leq x) = \frac{\Gamma(x+1, \mu)}{\Gamma(x+1)} \quad x = 0, 1, 2, \ldots.
\]

The survivor function on the support of \( X \) is

\[
S(x) = P(X \geq x) = \frac{\Gamma(x+1) - x\Gamma(x, \mu)}{x!} \quad x = 0, 1, 2, \ldots.
\]

The hazard function on the support of \( X \) is

\[
h(x) = \frac{f(x)}{S(x)} = \frac{\mu^x e^{-\mu}}{\Gamma(x+1) - x\Gamma(x, \mu)} \quad x = 0, 1, 2, \ldots.
\]

The cumulative hazard function on the support of \( X \) is

\[
H(x) = -\ln S(x) = -\ln \left[ \frac{\Gamma(x+1) - x\Gamma(x, \mu)}{x!} \right] \quad x = 0, 1, 2, \ldots.
\]
The inverse distribution function of $X$ is mathematically intractable but can run in APPL with statement at the bottom of the page.

The median, $m$, of $X$ is approximately (see Wikipedia site)

$$m \approx \left\lfloor \mu + 1/3 - 0.02/\mu \right\rfloor.$$

The moment generating function of $X$ is

$$M(t) = E\left[ e^{tX} \right] = e^{\mu(e^t-1)} \quad -\infty < t < \infty$$

The characteristic function of $X$ is

$$\phi(t) = E\left[ e^{itX} \right] = e^{\mu(e^{it}-1)} \quad -\infty < t < \infty$$

The population mean, variance, skewness, and kurtosis of $X$ are

$$E[X] = \mu \quad V[X] = \mu \quad E\left[ \left( \frac{X-\mu}{\sigma} \right)^3 \right] = \mu^{-1/2} \quad E\left[ \left( \frac{X-\mu}{\sigma} \right)^4 \right] = 3 + \mu^{-1}.$$  

**APPL verification**: The APPL statements

```appl
X := PoissonRV(mu);
CDF(X);
SF(X);
HF(X);
CHF(X);
IDF(X);
Mean(X);
Variance(X);
Skewness(X);
Kurtosis(X);
MGF(X);
```

verify the cumulative distribution function, survivor function, hazard function, cumulative hazard function, inverse distribution function, population mean, variance, skewness, kurtosis, and moment generating function.