Pascal distribution (from http://www.math.wm.edu/~leemis/chart/UDR/UDR.html)

The shorthand \( X \sim \text{Pascal}(n, p) \) is used to indicate that the random variable \( X \) has the Pascal distribution with positive integer parameter \( n \) and real parameter \( p \) satisfying \( 0 < p < 1 \). A Pascal random variable \( X \) has probability mass function

\[
f(x) = \binom{n-1+x}{x} p^n (1-p)^x \quad x = 0, 1, 2, \ldots.
\]

The Pascal distribution is also known as the negative binomial distribution. The Pascal distribution can be used to model the number of failures before the \( n \)th success in repeated mutually independent Bernoulli trials, each with probability of success \( p \). Applications include acceptance sampling in quality control and modeling demand for a product. The probability mass function for three different parameter settings is illustrated below.

The cumulative distribution function, survivor function, inverse distribution function, and hazard function of \( X \) are mathematically intractable. The moment generating function of \( X \) is

\[
M(t) = E[e^{tX}] = \left[ \frac{p}{1 - (1-p)e^t} \right]^n
\]

for \(|(1-p)e^t| < 1 \) or \( t < -\ln(1-p) \).

The population mean, variance, skewness, and kurtosis of \( X \) are

\[
E[X] = \frac{n(1-p)}{p} \quad V[X] = \frac{n(1-p)}{p^2}
\]

\[
E \left[ \left( \frac{X-\mu}{\sigma} \right)^3 \right] = \frac{2-p}{\sqrt{n(1-p)}}
\]

\[
E \left[ \left( \frac{X-\mu}{\sigma} \right)^4 \right] = \frac{p^2 - 6p - 3np + 3n + 6}{n(1-p)}
\]
**APPL verification:** The APPL statements

\[ X := \text{NegativeBinomialRV}(n, p); \]
\[ \text{MGF}(X); \]
\[ \text{Variance}(X); \]
\[ \text{Skewness}(X); \]
\[ \text{Kurtosis}(X); \]

verify the moment generating function, population variance, skewness, and kurtosis.