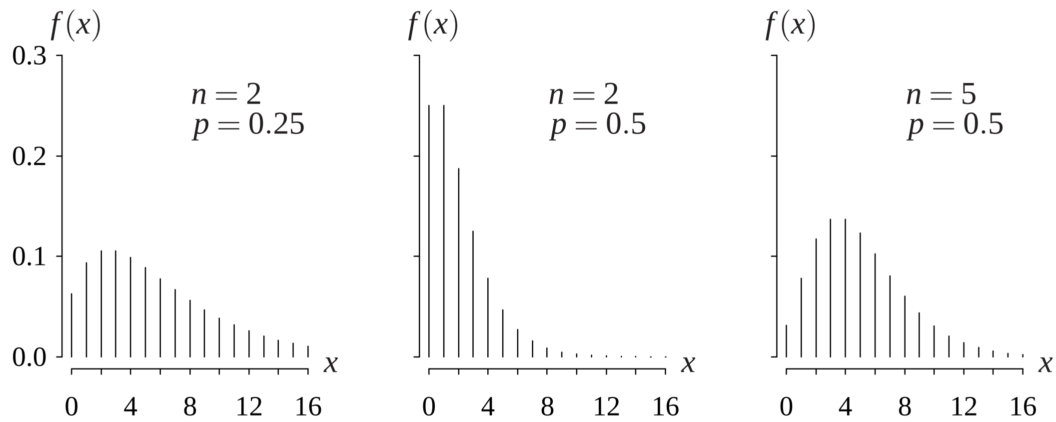
Pascal distribution (from http://www.math.wm.edu/~leemis/chart/UDR/UDR.html)

The shorthand $X \sim \text{Pascal}(n, p)$ is used to indicate that the random variable X has the Pascal distribution positive integer parameter n and real parameter p satisfying $0 < p < 1$. A Pascal random variable X has probability mass function

$$f(x) = \binom{n-1+x}{x} p^n (1-p)^x \qquad x = 0, 1, 2, \ldots.$$

The Pascal distribution is also known as the negative binomial distribution. The Pascal distribution can be used to model the number of failures before the nth success in repeated mutually independent Bernoulli trials, each with probability of success p. Applications include acceptance sampling in quality control and modeling demand for a product. The probability mass function for three different parameter settings is illustrated below.



The cumulative distribution function, survivor function, inverse distribution function, and hazard function of X are mathematically intractable. The moment generating function of X is

$$M(t) = E\left[e^{tX}\right] = \left[\frac{p}{1-(1-p)e^t}\right]^n$$

for $|(1-p)e^t| < 1$ or $t < -\ln(1-p)$.

The population mean, variance, skewness, and kurtosis of X are

$$E[X] = \frac{n(1-p)}{p} \qquad V[X] = \frac{n(1-p)}{p^2}$$

$$E\left[\left(\frac{X-\mu}{\sigma}\right)^3\right] = \frac{2-p}{\sqrt{n(1-p)}} \qquad E\left[\left(\frac{X-\mu}{\sigma}\right)^4\right] = \frac{p^2-6p-3np+3n+6}{n(1-p)}.$$

APPL verification: The APPL statements

```
X := NegativeBinomialRV(n, p);
MGF(X);
Variance(X);
Skewness(X);
Kurtosis(X);
```

verify the moment generating function, population variance, skewness, and kurtosis.