

Theorem Random variates from the Pareto(λ, κ) distribution can be generated in closed form by inversion.

Proof The Pareto distribution has probability density function

$$f(x) = \frac{\kappa \lambda^\kappa}{x^{\kappa+1}} \quad x \geq \lambda$$

and cumulative distribution function

$$F(x) = 1 - \left(\frac{\lambda}{x}\right)^\kappa \quad x \geq \lambda.$$

Equating the cumulative distribution function to u , where $0 < u < 1$ yields an inverse cumulative distribution function

$$F^{-1}(u) = \lambda(1 - u)^{-1/\kappa} \quad 0 < u < 1.$$

So a closed-form variate generation algorithm using inversion for the Pareto distribution is

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generate  $U \sim U(0, 1)$   
 $X \leftarrow \lambda(1 - U)^{-1/\kappa}$   
return( $X$ )
```