Theorem The minimum of n independent Pareto(λ, κ_i) random variables, where $i = 1, 2, \ldots, n$, is also a Pareto random variable with parameters λ and $\sum_{i=1}^{n} \kappa_i$.

Proof The cumulative distribution function of the Pareto random variable X is given by

$$\begin{aligned} F_X(x) &= \int_0^x \frac{\kappa\lambda^\kappa}{w^{\kappa+1}} dw \\ &= 1 - \left(\frac{\lambda}{x}\right)^\kappa \qquad\qquad x \geq \lambda. \end{aligned}$$

Let $Y = \min\{X_1, X_2, \ldots, X_n\}$. The cumulative distribution function of the minimum of n independent Pareto random variables is

$$\begin{aligned} F_Y(y) &= P(Y \leq y) \\ &= 1 - P(Y \geq y) \\ &= 1 - P(\min\{X_1, X_2, \ldots, X_n\} \geq y) \\ &= 1 - P(X_1 \geq y, X_2 \geq y, \ldots, X_n \geq y) \\ &= 1 - P(X_1 \geq y)P(X_2 \geq y) \ldots P(X_n \geq y) \\ &= 1 - \left(\frac{\lambda}{y}\right)^{\kappa_1} \left(\frac{\lambda}{y}\right)^{\kappa_2} \cdots \left(\frac{\lambda}{y}\right)^{\kappa_n} \\ &= 1 - \left(\frac{\lambda}{y}\right)^{\sum_{i=1}^{n} \kappa_i} \qquad\qquad y \geq \lambda. \end{aligned}$$

Since this has the same functional form of the cumulative distribution function of a Pareto random variable, the Pareto distribution has the minimum property.