**Theorem** The natural logarithm of a Pareto($\lambda, \kappa$) random variable minus the natural logarithm of $\lambda$ has the exponential distribution.

**Proof** Let the random variable $X$ have the Pareto distribution with probability density function

$$f_X(x) = \frac{\kappa \lambda^\kappa}{x^{\kappa+1}} \quad x > \lambda.$$ 

The transformation $Y = g(X) = \ln \left( \frac{X}{\lambda} \right)$ is a 1–1 transformation from $\mathcal{X} = \{x \mid x > \lambda\}$ to $\mathcal{Y} = \{y \mid y > 0\}$ with inverse $X = g^{-1}(Y) = \lambda e^Y$ and Jacobian

$$\frac{dX}{dY} = \lambda e^Y.$$ 

Therefore by the transformation technique, the probability density function of $Y$ is

$$f_Y(y) = f_X(g^{-1}(y)) \left| \frac{dx}{dy} \right|$$

$$= \frac{\kappa \lambda^\kappa}{(\lambda e^y)^{\kappa+1}} \lambda e^y$$

$$= \kappa e^{-y} \quad y > 0.$$ 

Letting $\kappa = 1/\alpha$,

$$f_Y(y) = \frac{1}{\alpha} e^{-y/\alpha} \quad y > 0,$$

which is the probability density function of the exponential distribution.