Theorem The natural logarithm of a Pareto(λ, κ) random variable minus the natural logarithm of λ has the exponential distribution.

Proof Let the random variable X have the Pareto distribution with probability density function

$$f_X(x) = \frac{\kappa\lambda^\kappa}{x^{\kappa+1}} \qquad x > \lambda.$$

The transformation $Y = g(X) = \ln\left(\frac{X}{\lambda}\right)$ is a 1–1 transformation from $\mathcal{X} = \{x \,|\, x > \lambda\}$ to $\mathcal{Y} = \{y \,|\, y > 0\}$ with inverse $X = g^{-1}(Y) = \lambda e^Y$ and Jacobian

$$\frac{dX}{dY} = \lambda e^Y.$$

Therefore by the transformation technique, the probability density function of Y is

$$\begin{aligned} f_Y(y) &= f_X(g^{-1}(y)) \left|\frac{dx}{dy}\right| \\ &= \frac{\kappa\lambda^\kappa}{(\lambda e^y)^{\kappa+1}} |\lambda e^y| \\ &= \kappa e^{-y\kappa} \qquad y > 0. \end{aligned}$$

Letting $\kappa = 1/\alpha$,

$$f_Y(y) = \frac{1}{\alpha} e^{-y/\alpha} \qquad y > 0,$$

which is the probability density function of the exponential distribution.