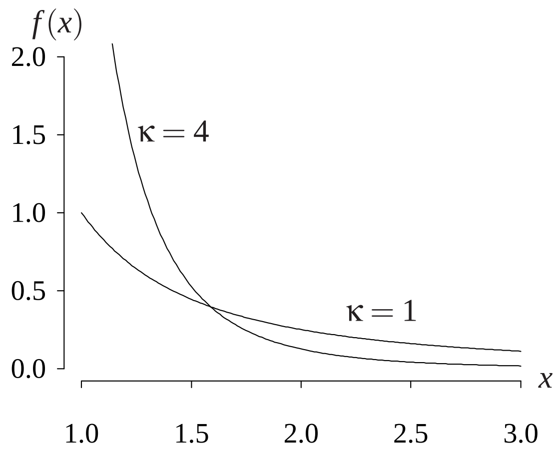
Pareto distribution (from http://www.math.wm.edu/~leemis/chart/UDR/UDR.html)

The shorthand $X \sim \text{Pareto}(\lambda, \kappa)$ is used to indicate that the random variable X has the Pareto distribution with parameters λ and κ. A Pareto random variable X with positive parameters λ and κ has probability density function

$$f(x) = \frac{\kappa \lambda^{\kappa}}{x^{\kappa+1}} \qquad x > \lambda.$$

The Pareto distribution has traditionally been used to model the distribution of income, where λ is a minimum wage and κ models the distribution of the income. The Pareto distribution can also be used to model the lifetime of an object with a warranty period λ or the duration of a strike with minimum duration λ. The probability density function with $\lambda = 1$ and two different values of κ is illustrated below.



The cumulative distribution function on the support of X is

$$F(x) = P(X \le x) = 1 - \left(\frac{\lambda}{x}\right)^{\kappa} \qquad x > \lambda.$$

The survivor function on the support of X is

$$S(x) = P(X \ge x) = \left(\frac{\lambda}{x}\right)^{\kappa} \qquad x > \lambda.$$

The hazard function on the support of X is

$$h(x) = \frac{f(x)}{S(x)} = \frac{\kappa}{x} \qquad x > \lambda.$$

The cumulative hazard function on the support of X is

$$H(x) = -\ln S(x) = -\kappa(\ln \lambda - \ln x) \qquad x > \lambda.$$

The inverse distribution function of X is

$$F^{-1}(u) = \lambda(1-u)^{-1/\kappa} \qquad\qquad 0 < u < 1.$$

The median of X is

$$\lambda 2^{1/\kappa}.$$

The moment generating function of X is

$$M(t) = E\left[e^{tX}\right] = \kappa(-\lambda t)^{\kappa}\Gamma(-\kappa, -\lambda t) \qquad\qquad -\infty < t < \infty.$$

The characteristic function of X is

$$\phi(t) = E\left[e^{itX}\right] = \kappa(-\lambda it)^{\kappa}\Gamma(-\kappa, -\lambda it) \qquad\qquad -\infty < t < \infty.$$

The population mean, variance, skewness, and kurtosis of X are

$$E[X] = \frac{\kappa\lambda}{\kappa-1} \text{ for } \kappa > 1 \qquad\qquad V[X] = \frac{\kappa\lambda^2}{(\kappa-1)^2(\kappa-2)} \text{ for } \kappa > 2$$

$$E\left[\left(\frac{X-\mu}{\sigma}\right)^3\right] = 2\,\frac{\kappa+1}{\kappa-3}\sqrt{\frac{\kappa-2}{\kappa}} \text{ for } \kappa > 3 \qquad\qquad E\left[\left(\frac{X-\mu}{\sigma}\right)^4\right] = \frac{3(\kappa-2)(3\kappa^3+\kappa+2)}{\kappa(\kappa-3)(\kappa-4)} \text{ for } \kappa > 4.$$

APPL verification: The APPL statements

```
X := ParetoRV(lambda, kappa);
CDF(X);
SF(X);
HF(X);
CHF(X):
IDF(X);
assume(kappa > 1);
Mean(X);
assume(kappa > 2);
Variance(X);
assume(kappa > 3);
Skewness(X);
assume(kappa > 4);
Kurtosis(X);
MGF(X);
```

verify the cumulative distribution function, survivor function, hazard function, cumulative hazard function, and inverse distribution function. Some of the population moments are expressed as limits.