Theorem If $X \sim N(\mu, \sigma^2)$ and $\mu = 0$, $\sigma^2 = 1$, then X has the standard normal distribution.

Proof The probability density function of the normal distribution is

$$f_X(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \qquad -\infty < x < \infty.$$

Substituting $\mu = 0$ and $\sigma = 1$ yields

$$f_X(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2} \qquad -\infty < x < \infty,$$

which is the probability density function of the standard normal distribution.

APPL verification: The APPL statements

```
X := NormalRV(mu, sigma);
subs({mu = 0, sigma = 1}, X[1][1](x));
```

yield the probability density function of a $N(0, 1)$ random variable.