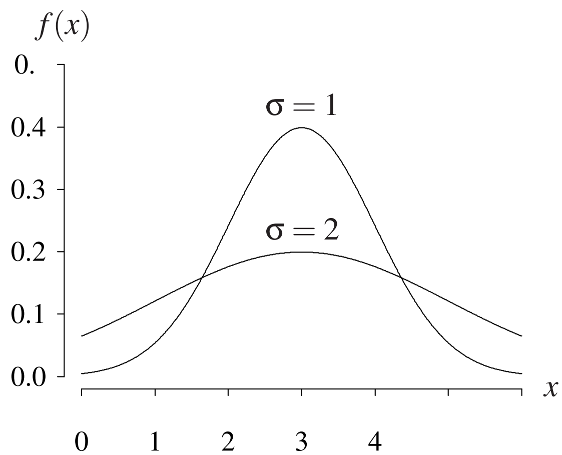
Normal distribution (from http://www.math.wm.edu/~leemis/chart/UDR/UDR.html)
The shorthand $X \sim \mathrm{N}(\mu, \sigma^2)$ is used to indicate that the random variable X has the normal distribution with parameters μ and σ^2. A normal random variable X with mean μ and variance σ^2 has probability density function

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \qquad -\infty < x < \infty,$$

for $-\infty < \mu < \infty$ and $\sigma > 0$. The normal distribution can be used for modeling adult heights, newborn baby weights, ball bearing diameters, etc. The normal distribution can be used to approximate the binomial distribution when n is large and p is close to $1/2$. The normal distribution can also be used to approximate the Poisson distribution when n is large and p is small. The central limit theorem indicates that the normal distribution is useful for modeling random variables that can be thought of as a sum of several independent random variables. The probability density function for $\mu = 3$ and two different values of σ is illustrated below.



The cumulative distribution function on the support of X is

$$F(x) = P(X \le x) = \frac{\mathrm{erf}\left(\frac{x-\mu}{\sqrt{2}\sigma}\right) + 1}{2} \qquad -\infty < x < \infty,$$

where

$$\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt \qquad x > 0$$

and $\mathrm{erf}(-x) = -\mathrm{erf}(x)$. The survivor function on the support of X is

$$S(x) = P(X \ge x) = \frac{1 - \mathrm{erf}\left(\frac{x-\mu}{\sqrt{2}\sigma}\right)}{2} \qquad -\infty < x < \infty.$$

The hazard function on the support of X is

$$h(x) = \frac{f(x)}{S(x)} = -\frac{e^{-\frac{(x-\mu)^2}{2\sigma^2}}\sqrt{2}}{\sigma\sqrt{\pi}(\text{erf}(\frac{x-\mu}{\sqrt{2}\sigma})-1)} \qquad -\infty < x < \infty.$$

The hazard function can be difficult to calculate for large values of x because the survivor function $S(x)$ and the probability density function $f(x)$ are small. Details and a fix in R are given at http://stackoverflow.com/questions/39510213/calculating-hazard-function-in-r-for-the-standard-normal-distribution.
The cumulative hazard function on the support of X is mathematically intractable.

The inverse distribution function of X is

$$F^{-1}(u) = \mu + \sigma\sqrt{2}\,\text{erf}^{-1}(2u-1) \qquad 0 < u < 1.$$

The median of X is μ.
The moment generating function of X is

$$M(t) = E\left[e^{tX}\right] = e^{t(t\sigma^2+2\mu)/2} \qquad -\infty < t < \infty.$$

The characteristic function of X is

$$\phi(t) = E\left[e^{itX}\right] = e^{t(-t\sigma^2+2i\mu)/2} \qquad -\infty < t < \infty.$$

The population mean, variance, skewness, and kurtosis of X are

$$E[X] = \mu \qquad V[X] = \sigma^2 \qquad E\left[\left(\frac{X-\mu}{\sigma}\right)^3\right] = 0 \qquad E\left[\left(\frac{X-\mu}{\sigma}\right)^4\right] = 3.$$

APPL verification: The APPL statements

```
X := NormalRV(mu, sigma);
CDF(X);
SF(X);
HF(X);
IDF(X);
Mean(X);
Variance(X);
Skewness(X);
Kurtosis(X);
MGF(X);
```

verify the cumulative distribution function, survivor function, hazard function, inverse distribution function, population mean, variance, skewness, kurtosis, and moment generating function.