Theorem The t distribution is a special case of the noncentral t distribution when $\delta = 0$.

Proof The noncentral t distribution has probability density function

$$f(x) = \frac{n^{n/2} e^{-\delta^2/2}}{\sqrt{\pi}\Gamma(n/2)(n+x^2)^{(n+1)/2}} \sum_{i=0}^{\infty} \frac{\Gamma[(n+i+1)/2]}{i!} \left(\frac{x\delta\sqrt{2}}{\sqrt{n+x^2}} \right)^i \qquad -\infty < x < \infty.$$

When $\delta = 0$, this reduces to

$$\begin{aligned} f(x) &= \frac{n^{n/2}\Gamma[(n+1)/2]}{\sqrt{\pi}\Gamma(n/2)(n+x^2)^{(n+1)/2}} \\ &= \frac{n^{n/2}\Gamma[(n+1)/2]}{\sqrt{\pi}\Gamma(n/2)(x^2/n+1)^{(n+1)/2} n^{(n+1)/2}} \\ &= \frac{n^{n/2}\Gamma[(n+1)/2]}{\sqrt{n\pi}\Gamma(n/2)(x^2/n+1)^{(n+1)/2} n^{n/2}} \\ &= \frac{\Gamma[(n+1)/2]}{(n\pi)^{1/2}\Gamma(n/2)(x^2/n+1)^{(n+1)/2}} \qquad -\infty < x < \infty. \end{aligned}$$

which is the probability density function of the t distribution. In the first step, the elements in the sum all become zero except for $i = 0$.

APPL verification: The APPL statements

```
X := [[x -> n ^ (n / 2) * exp(-(delta ^ 2) / 2) / (sqrt(pi) * (GAMMA(n / 2)) *
           (n + x ^ 2) ^ ((n + 1) / 2)) * sum(((GAMMA((n + i + 1) / 2) / (i!)) *
           (x * delta * sqrt(2) / (sqrt(n + x ^ 2))) ^ i), i = 0 .. infinity)],
           [-infinity, infinity], ["Continuous", "PDF"]];
limit(X[1][1](x), delta = 0);
TRV(n);
```

confirm that the t distribution is a special case of the noncentral t distribution when $\delta = 0$.