Theorem The chi-square distribution is a special case of the noncentral chi-square distribution when $\delta = 0$.

Proof The noncentral chi-square has probability density function

$$f(x) = \sum_{k=0}^{\infty} \frac{e^{-\delta/2}(\frac{\delta}{2})^k}{k!} \cdot \frac{e^{-x/2} x^{\frac{n+2k}{2}-1}}{2^{\frac{n+2k}{2}} \Gamma(\frac{n+2k}{2})}, \qquad x > 0$$

When $\delta = 0$, this reduces to

$$\begin{aligned} f(x) &= \sum_{k=0}^{\infty} \frac{e^{-0/2}(\frac{0}{2})^k}{k!} \cdot \frac{e^{-x/2} x^{\frac{n+2k}{2}-1}}{2^{\frac{n+2k}{2}} \Gamma(\frac{n+2k}{2})} \\ &= \frac{e^{-(x/2)} x^{(n/2)-1}}{2^{(n/2)} \Gamma(n/2)} \qquad x > 0 \end{aligned}$$

which is the probability density function of the chi-square distribution with n degrees of freedom.